TECHNICAL REPORT M-71-5

PERFORMANCE OF SOILS UNDER TRACK LOADS

Report 3

TRACK MOBILITY NUMBER FOR COARSE-GRAINED SOILS

by

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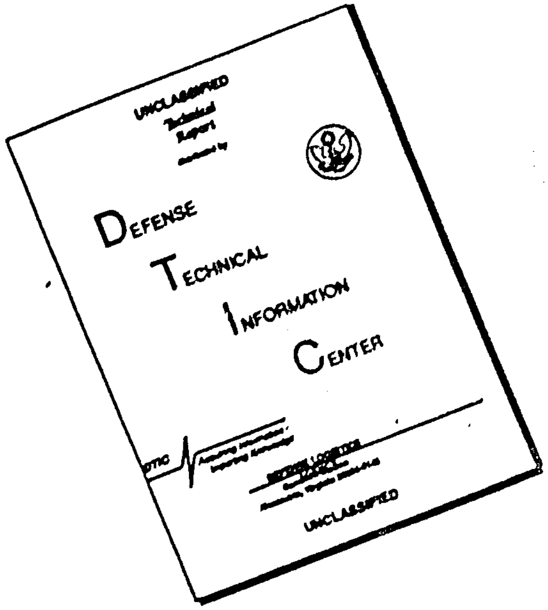

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Dimensional analysis was used to produce a listing of 28 dimensionless Pi terms from which to develop a comprehensive description of straight-line, level-ground, single-track performance in soil. Five of these terms--P/W, P_T^m/W, M/Wr, z/ℓ, and θ--include dependent performance variables: pull (P), towed force (P_T^m), torque (M), sinkage (z), and trim angle (θ), respectively. The remaining 23 Pi terms contain only independent variables; 15 of these terms are needed to describe track operation in air-dry sand. Among

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20. ABSTRACT(Continued)

the independent variables included in these 15 terms, seven were determined from laboratory model-track tests to have significantly more effect than the rest on track pull at 20 percent slip (P_{20}): load (W), sand penetration resistance gradient (G), track width (b), nominal track-ground contact length (ℓ), hard-surface road-bogie deflection (Δ), horizontal at-rest center of gravity (RCG_h), and spacing between road wheels (s).

A single dimensionless prediction term, sand-track mobility number

$$N_s = \frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2} \cdot \left(\frac{d}{\ell/2}\right)^n$$

was developed that takes all seven of these independent variables into account. ~~The ratio~~ of Δ to maximum deflection Δ_{max} of the road bogies is directly related to $\frac{W}{W_{max}}$, where W_{max} is the load that produces Δ_{max}. d is the distance from the center of the rear road wheel to the track horizontal at-rest center of gravity, and $\frac{d}{\ell/2}$ is a useful dimensionless surrogate for RCG_h. (Exponent n = 1, 3/2, or 1/2 depending on whether the RCG_h is at, rearward of, or forward of, respectively, the track geometric center line.) Road-wheel spacing [redacted] was found to influence track performance significantly only for those values of [redacted] road-wheel diameter larger than those ordinarily used or recommended for prototype tracked vehicles.

Track pull is increased slightly by (a) locating the drive sprocket at the rear rather than the front of the track; (b) maintaining high tension in the track belt; and (c) using a decreasing, rather than an increasing, front-to-rear pattern of road-bogie cylinder pressure, i.e. by decreasing the ability of the road bogies to resist deflection from front to rear of track. Torque is not closely related to the sand-track mobility number, but can be predicted from its relation to load times drive-sprocket radius (Wr) at the self-propelled, the maximum-tractive-efficiency (TE_{max}), and the 20 percent slip points. Sand-track mobility number sub

A comprehensive set of relations between N_s and performance terms P/W, slip, tractive efficiency (i.e., output power/input power), z/ℓ, and θ is developed for the towed, self-propelled, TE_{max}, and 20 percent slip conditions. These relations can be used to predict the in-sand performance of a given track for these (or intermediate) track performance levels; or, reversing the order they can be used to select or design a loaded track to satisfy a particular in-sand performance requirement. Though developed from level-ground tests of a single track, the pull/load versus N_s relations can easily be extrapolated to slope-climbing or vehicle-towing situations. Laboratory tests with four full-size tracked vehicles indicate that the model-developed sand-track mobility number can be used to described prototype vehicle performance.

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PREFACE

This report comprises a study of results from laboratory tests of single model tracks in sand conducted at the U. S. Army Engineer Waterways Experiment Station (WES) as a part of the vehicle mobility research program under Department of the Army Project 1G662601AH91, "Track and Automotive Technology-Validation and Extension of Mobility Evaluation Methodology," Task A046, Subtask 06, "Development of Track-Soil Performance Model Based on Numerics," under the sponsorship and guidance of the Research, Development, and Engineering Directorate, U. S. Army Materiel Development and Readiness Command.

Testing to determine the performance of soils under track loads is being conducted by personnel of the Mobility Research and Methodology Branch (MRMB) and the Mobility Investigations Branch (MIB) of the Mobility Systems Division (MSD), Mobility and Environmental Systems Laboratory (MESL), WES, under the general supervision of Mr. W. G. Shockley, Chief of the MESL, and Mr. A. A. Rula, Chief of the MSD, and under the direct supervision of Mr. C. J. Nuttall, Jr., Chief of the MRMB and Mr. E. S. Rush, Chief of the MIB. Personnel of the Data Handling Branch (DHB), MESL, Mr. J. L. Smith, Chief, converted the data examined herein from analog to digital form. This report was prepared by Mr. G. W. Turnage, MRMB.

BG E. D. Peixotto, CE, and COL G. H. Hilt, CE, were Directors of the WES during conduct of this study and preparation of this report. Mr. F. R. Brown was Technical Director.

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CONVERSION FACTORS, METRIC (SI) TO U. S. CUSTOMARY
UNITS OF MEASUREMENT

Metric (SI) units of measurement used in this report can be converted to U. S. customary units as follows:

Multiply	By	To Obtain
millimetres	0.03937007	inches
metres	3.280839	feet
centimetres	0.3937007	inches
centimetres per second	0.3937007	inches per second
square centimetres	0.1550	square inches
kilonewtons	224.8089	pounds (force)
meganewtons per cubic metre	3.684	pounds (force) per cubic inch
kilopascals	0.1450377	pounds (force) per square inch
newtons	0.2248089	pounds (force)
metre-newtons	0.7375621	foot-pounds
metres per second	3.280839	feet per second
grams per cubic centimetre	62.42797	pounds (mass) per cubic foot

PERFORMANCE OF SOILS UNDER TRACK LOADS

TRACK MOBILITY NUMBER FOR COARSE-GRAINED SOILS

PART I: INTRODUCTION

Background

1. A major effort in the mobility research at the U. S. Army Engineer Waterways Experiment Station (WES) is aimed at improving the on-ground mobility of military vehicles. An important goal of this research is to develop a comprehensive mathematical model that can describe accurately the performance of tracked vehicles operating off-road. WES is accomplishing this objective in a three-stage program. First-stage efforts concentrate on analysis of data from a systematic program of laboratory tests of a versatile, thoroughly instrumented model track system. The model-track test program is being complemented by tests of full-scale tracked vehicles, first in the laboratory and then in the field, in second- and third-stage programs designed to verify (or modify, as needed) the laboratory-developed mathematical model and to extend its range of application.

2. Report 1 of this series[1] presented in detail (a) definitions of pertinent soil and track descriptors, (b) a description of the WES model track, laboratory equipment, and test techniques, and (c) an outline of the long-range WES track test program. In Report 2[2] a statistical (Plackett-Burman) design was used to determine those independent track and sand variables that have most influence on track pull at 20 percent slip (P_{20}), and a basic-variable prediction term

$$N_{BV} = \frac{G(b\ell)^{3/2}}{W} \qquad (1)$$

where

G = sand penetration resistance gradient

b = track width

ℓ = track length

W = load on a single track

was developed that can be used to predict P_{20} for a track operating in a straight line (i.e. without maneuvering) in level, flat test sections of air-dry desert sand.

Purpose

3. The purpose of the overall model-track test program is to develop a comprehensive methodology (in this case in dimensionless terms) that can be used to describe soil-track vehicle interations. The purposes of the work reported herein were to:

- a. Expand the basic-variable dimensionless prediction term (equation 1) to include additional important independent variables.
- b. Relate the expanded term from a to several dependent performance terms from laboratory tests of the model track in two different sands.
- c. Validate the usefulness of the expanded prediction term to describe straight-line, level-ground, prototype tracked vehicle performance in air-dry sand.

Scope

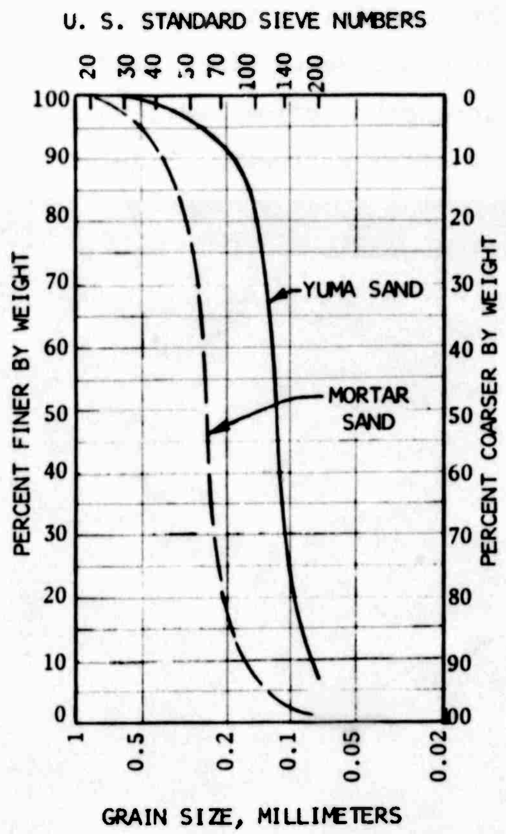
4. In the overall model-track test program, all sand and track variables thought to have a reasonable chance of significantly influencing straight-line track performance in level sections of air-dry sand are considered. Herein, the need and utility of expanding the basic-variable prediction term to include additional independent variables was examined. The analysis concentrated on the 20-percent slip (or near-maximum-pull) condition, but attention was also given to three other performance levels, the towed, self-propelled, and maximum-tractive-efficiency conditions. Measurements of track performance included towed force, sinkage, trim angle, pull, and torque. The ranges of values of the independent sand and track variables considered herein, when expressed in the dimensionless prediction term forms described under "Analysis of Data," cover essentially the full range of prediction term values important to mobility that are encountered by tracked vehicles in the field. Laboratory tests were conducted with four full-size tracked vehicles to validate the performance prediction relations developed from tests of a single model track.

PART II: TEST PROGRAM

Soils and Their Preparation

Test soils

5. The primary test soil was a fine sand taken from active desert dunes near Yuma, Arizona (Yuma sand); the secondary test material was a coarser sand obtained from a riverbed south of Vicksburg, Mississippi (mortar sand). The Yuma and mortar sands are uniformly graded, subangular, and classed as SP-SM and SP, respectively, according to the Unified Soil Classification System. Grain-size distribution curves and soil property data for the two test soils are presented in Figure 1.



SAND	CLASSIFICATION	DENSITY, gm/cc MAX	MIN
YUMA	SP-SM	1.68	1.40
MORTAR	SP	1.70	1.41

Figure 1. Gradation and classification of Yuma and mortar sands

Each sand was tested in an air-dry state, with moisture content nearly constant at about 0.5 percent.

Soil preparation

6. Most tests reported herein were conducted in a test pit 3.5 m wide,* 1.9 m deep, and 54.9 m long, with a 30-m length normally used for testing. A number of early tests were made in a soil bin 1.6 m wide, 0.8 m deep, and 41.2 m long. In each of the soil container arrangements, the sand was thoroughly harrowed between tests to at least the 40-cm depth and to a width at least 30 cm greater than that of the track width subsequently tested. Preparation of a low-strength test section was completed simply by leveling the sand surface with a screed board. Intermediate- to high-strength test sections were produced by harrowing, compacting with a vibratory skid unit (comprised of an electric vibrator mounted on a steel baseplate 86 cm wide), and then leveling. In the test pit, very high strength sections were obtained by using a heavy pneumatic-tired roller after harrowing (Figure 2). Each of these

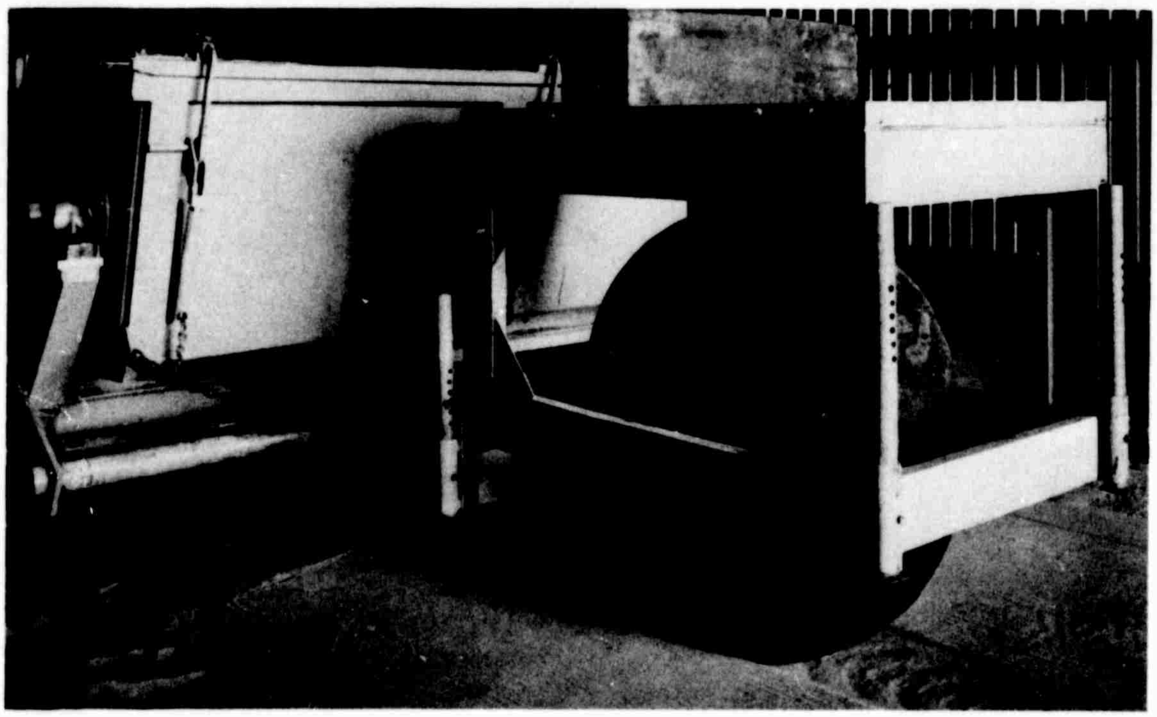


Figure 2. Compacting sand with pneumatic tire prior to traffic test

* A table of factors for converting metric (SI) units of measurement to U. S. customary units is given on page 4.

procedures produced uniform test sections in which cone penetration resistance C_s* increased in near-linear fashion to approximately the 30-cm depth. Representative profiles, showing three different strength levels, are presented in Figure 3. Strength of the air-dry, essentially cohesionless test soils is characterized herein by the slope of the cone penetration resistance versus depth curve, termed penetration resistance gradient G. Appendix A describes how sand test profiles were demonstated to be adequate for testing the WES model track at widths up to 61.0 cm. Parameter G is closely related to soil dry density for purely frictional soils[3] (Figure 4).

Test Equipment

7. Two dynamometer carriages were used in which the model tracks were mounted for testing: one intermediate-scale and the other large-scale (Figure 5)[1]. Single tracks of fairly large scale (about one-fourth to one-half the size of most conventional tracks) were tested. Figures and narrative descriptions in Reports 1 and 2 of this series illustrate the versatility of the model track system relative to its loading and suspension systems and to adjustments that can be made to various of its track geometry measurements.[1,2]

8. Except for some of the earliest tests, the model tracks were tested in the large-scale dynamometer carriage-soil pit system. The single-track test rig used in the large-scale carriage allows control of 22 independent variables considered to provide a reasonably comprehensive description of the soil-track system over the range of values listed in Table 1.

Test Techniques

9. No matter what test technique is used, two of the principal track performance relations are those of pull versus slip and torque versus slip. Representative curves in Figure 6 illustrate that from

* See Appendix B for expanded definitions of terms.

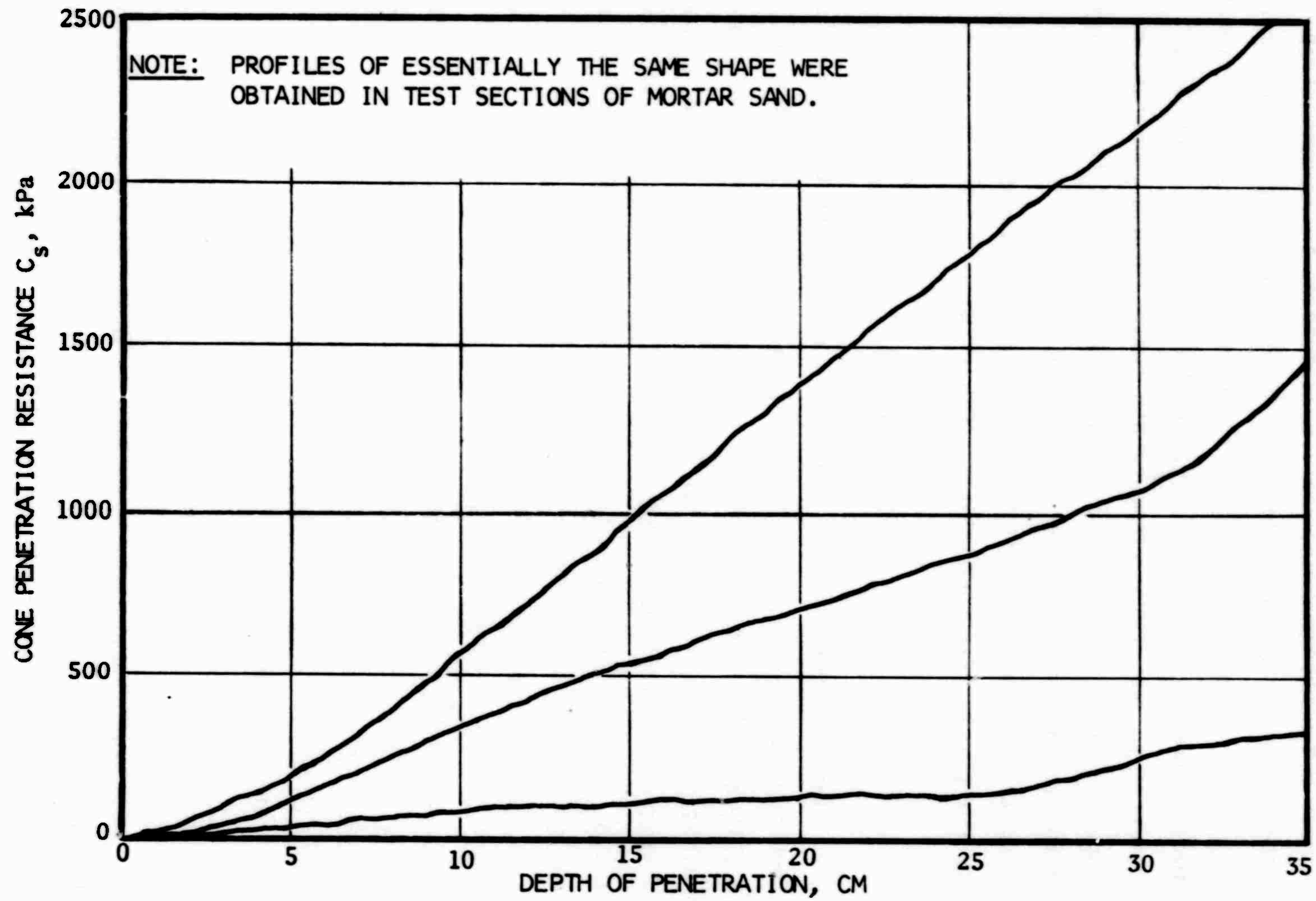


Figure 3. Representative cone penetration resistance versus depth profiles for air-dry Yuma sand

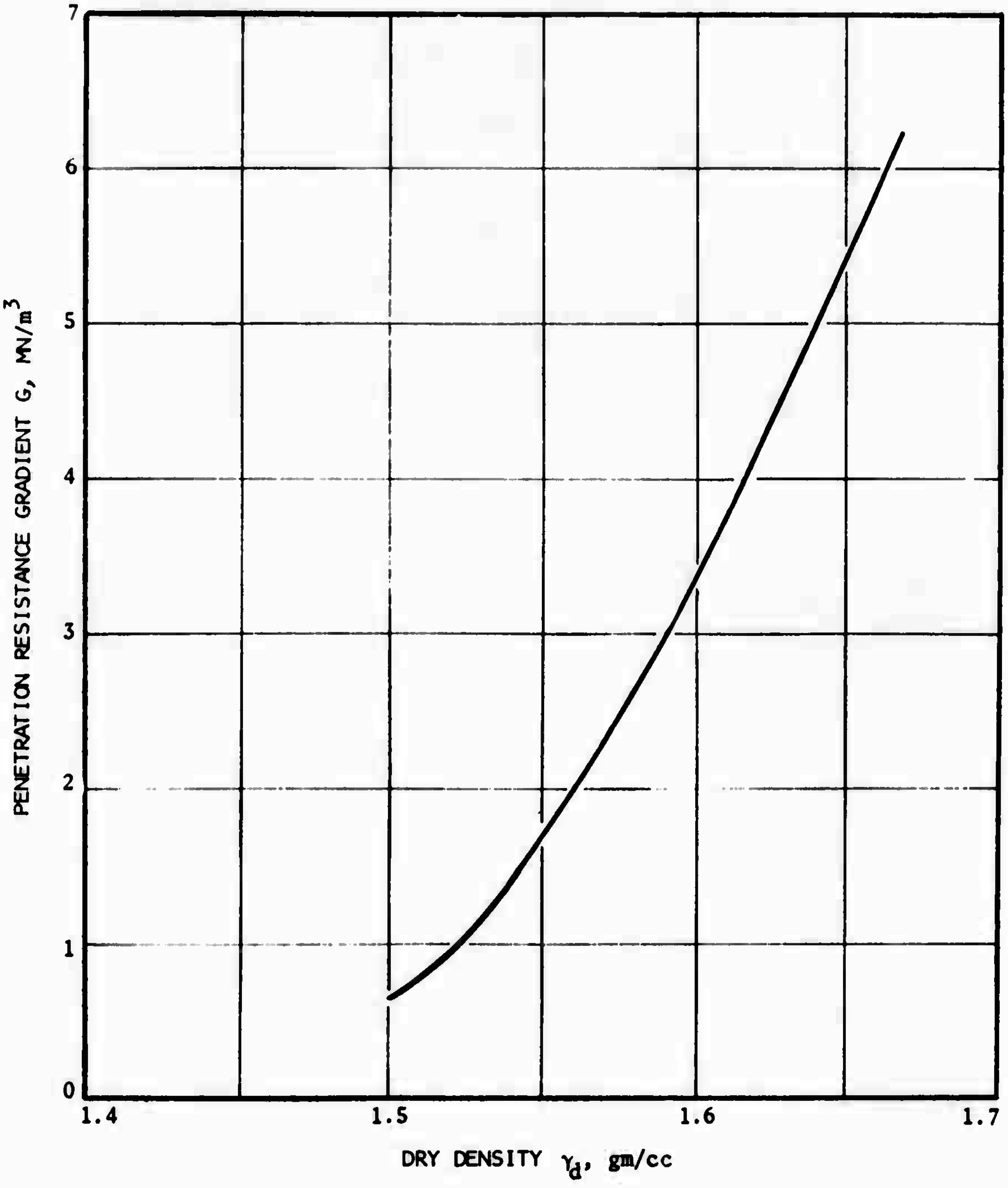
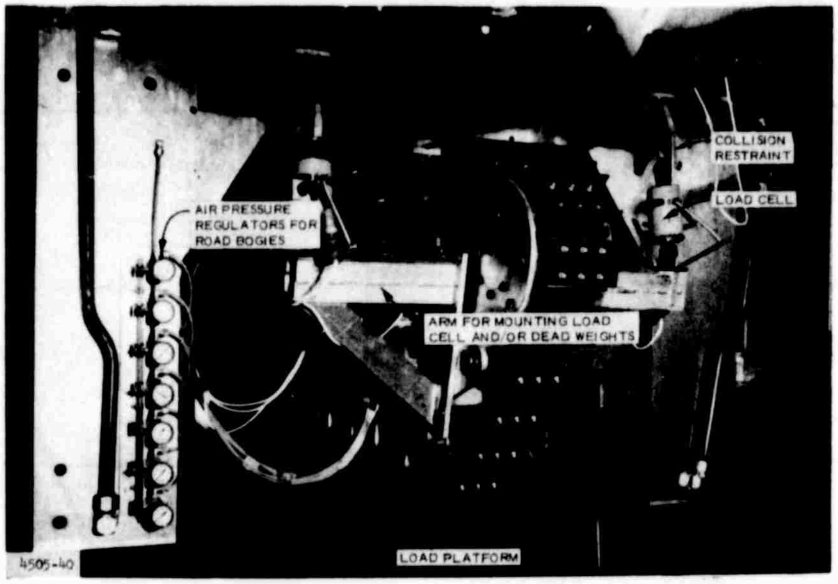
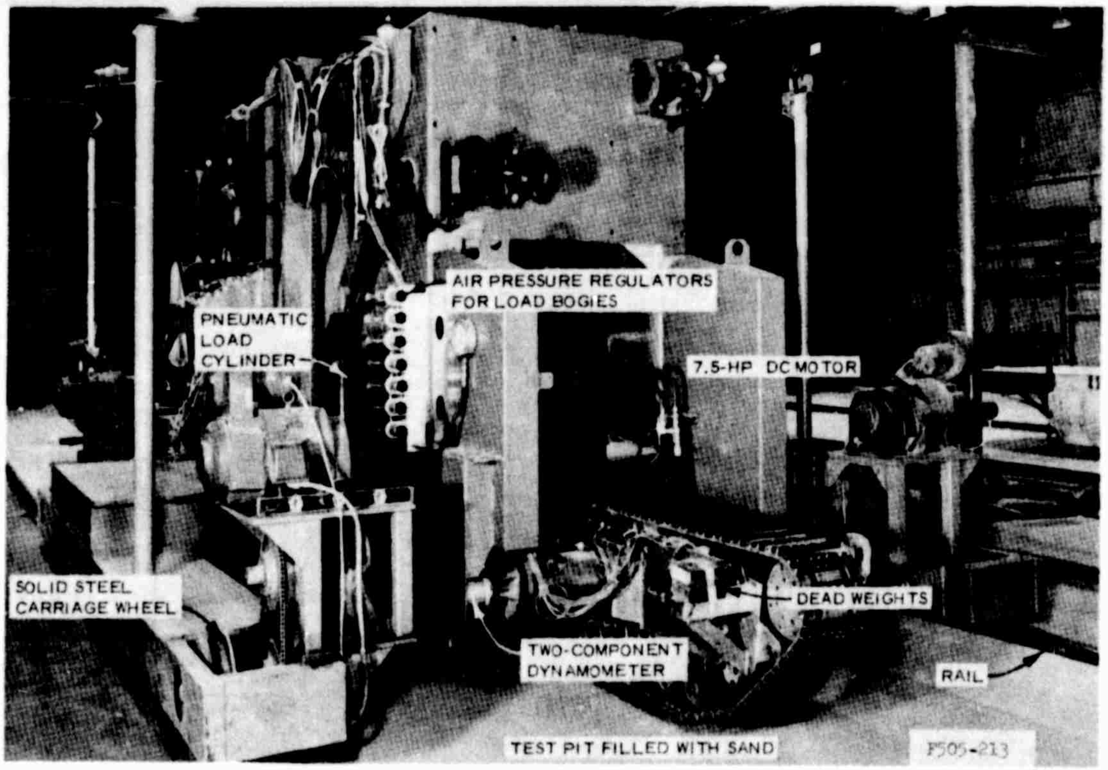


Figure 4. Relation between dry density and G for air-dry Yuma sand (adapted from Reference 3)



a. 15.2- by 61.0-cm track mounted in intermediate-scale test carriage



b. 30.5- by 121.9-cm track mounted in large-scale test carriage

Figure 5. WES model track in (a) intermediate- and (b) large-scale dynamometer carriage-soil container systems

these relations three performance levels of particular interest can be defined: the towed point (zero torque input, negative pull output, negative slip); the self-propelled point (positive torque input, zero pull output, small positive slip); and the 20 percent slip point (a nominal positive slip level at which near-maximum track pull usually occurs).

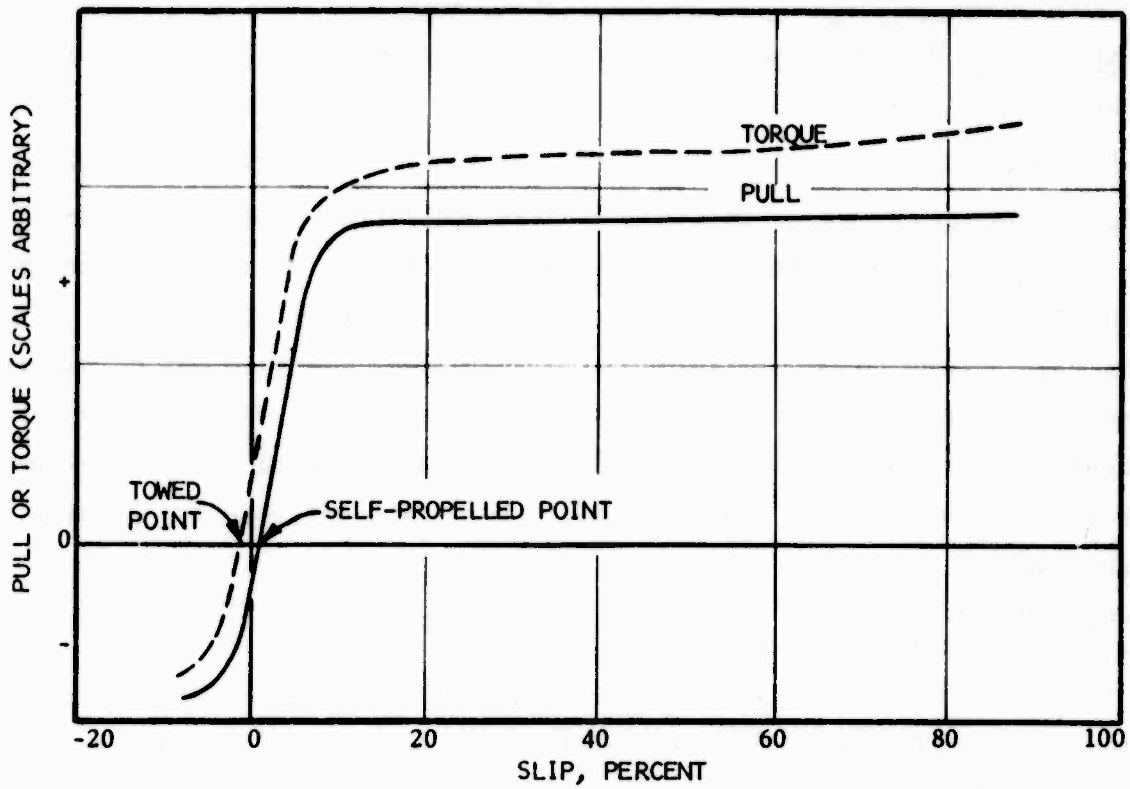


Figure 6. Representative pull-slip and torque-slip curves for tracks in air-dry sand

10. With the WES model track system, slip and load can each be programmed to vary or to remain constant as trim angle either assumes its own value or remains mechanically restrained at a preset value. In tabular form, the test technique options that result from these capabilities are:

	Load			
	Constant		Programmed	
	Trim Angle			
Slip	Unrestrained	Restrained	Unrestrained	Restrained
Constant	1	2	3	4
Programmed	5	6	7	8

One of the main concerns in using a given model-track test technique is that it produces results equivalent to those of the track of a prototype vehicle. In real-world situations, the load on a track may be considered constant, track attitude is unrestrained, and slip may be either near-constant or quite variable. These conditions are described by test technique options 1 and 5 above. More information on these two options, as well as on the other six tabulated above, is given in the following paragraphs.

Constant- and
programmed-slip tests

11. Constant-slip testing is produced when a given slip value is introduced by mechanically maintaining track theoretical and actual speeds at preselected values. Near-constant slip also results from a towed test or a constant-pull test in which the slip value, although not mechanically maintained, varies so slightly that for practical purposes it can be considered constant.

12. In a programmed-increasing-slip test, theoretical velocity (V_t) is held constant and actual velocity (V_a) is slowed at a programmed, uniform rate to cause the track to pass through the towed condition (torque = 0), the zero slip condition $(V_a = V_t)$, the self-propelled condition (pull = 0), etc., as slip is progressively increased to 100 percent $(V_a = 0)$. In Figure 6, pull increases rapidly with slip

from the towed condition (at a small negative slip value) to a positive slip value usually well under 20 percent, and continues to increase very slowly beyond that point. For a given programmed-increasing-slip test, the torque-slip curve is similar in shape to the pull-slip curve except that in the range of positive slip values where pull increases very slowly in near-linear fashion, torque increases in a more curvilinear pattern that reflects a generally larger percentage increase in the value of torque.

13. Because of the rapid increase in pull and torque values in about the -10 to +10 percent slip range of a programmed-increasing-slip test, more data scatter generally results in relations based on information extracted at a particular performance level within this range than would be produced from tests run under steady-state conditions (i.e. constant-slip, towed, or constant-pull tests). However, comparisons of track performance (pull, torque, sinkage, trim angle, etc.) from a number of paired tests, each alike in all respects except that one used test technique option 1 (paragraph 10) and the other option 5, showed that essentially the same test results are produced by the two options. Thus, it was possible to take advantage of the fact that far more information is yielded per test in programmed- than in constant-slip tests. Data from programmed-increasing-slip tests at the towed, the self-propelled, the 20-percent-slip, and the maximum-tractive-efficiency* points are analyzed in Part IV of this report.

Programmed-load and restrained-trim-angle tests

14. The programmed-load technique was used only in the large-scale test system. There, load is applied pneumatically and can be programmed to increase at a rate of approximately 1.0 kN per metre of carriage travel with the carriage moving at normal test speed (0.6 m/sec). Thus, the total load range of the model track system can easily be covered in the 30-m test length normally available. This technique appeared promising at its inception, but it has since been discarded for tests in sand (eliminating options 3, 4, 7, and 8 in paragraph 10)

* Maximum-tractive-efficiency, TE_{max} , is defined in paragraph 80.

because results from its use have generally not matched those from steady-state testing (i.e. either towed, constant-slip, or constant-load tests).

15. The restrained-time-angle constant-load technique (options 2 and 6) was generally avoided because test results associated with this artificial method of maintaining a track's trim angle are considered less useful than those produced by influencing the track's trim angle through control of the track's at-rest center of gravity (RCG). Data are examined herein, however, from a few restrained-trim-angle tests conducted in the intermediate-scale test systems. There, the clearance between the top of the track and the carriage tow cable was so small (only about 15 cm when the track was level at zero sinkage) that two restraints had to be constructed, one on each side of the test carriage (Figure 5a), to prevent the top of the track from striking the cable during the course of a test. Two load cells mounted on arms built on either side of the track engaged these restraints just prior to cable-track impact and measured total restraint force. Most tests reported herein were conducted in the large-scale dynamometer-soil pit system; of these only four (whose results are considered in paragraphs 34 and 35) were conducted with the trim angle restrained. For these four tests, a metal yoke composed of a load cell with roller bearings mounted on each end was mounted vertically between two horizontally aligned members. One member was projected from the inner carriage frame and the other from a point near the front of the track. For each restrained-trim-angle test conducted in either the intermediate- or the large-scale dynamometer carriages, total restraining force was treated as a track "component weight" in describing the force system acting on the track by the method discussed in paragraph 47.

16. Overall, then, the considerations above led to the use of test technique options 1 and 5 in the large majority of the WES model-track tests. Subsequent analysis in this report demonstrates that these options produced test results that followed well-defined trends and that, when considered in the proper dimensionless format, corresponded closely with prototype tracked vehicle test results.

PART III: DIMENSIONAL FRAMEWORK

Background

17. Markwick[4] in 1944 introduced dimensional analysis as a means of studying soil-vehicle relations when he presented a brief analysis of the bearing capacity of soft soils under tracked vehicles. A number of investigators have used dimensional analysis in the study of soil-tire relations, and Nuttall,[5] in particular, has advocated using this technique to study the soil-track system. To date, however, an analysis based on this approach has not been carried to completion for tracks operating in soil. Report 2 of this series[2] made a start in this effort.

18. In Report 2, results of tests based on Plackett-Burman (statistical) designs were analyzed to identify those independent sand and track variables that have greatest influence on P_{20} (track pull at 20 percent slip) in Yuma sand. All but six of the 22 variables listed in Table 1 were considered. Horizontal location of the at-rest center of gravity (RCG_h) was maintained at the geometric center line; RCG_v was maintained from zero to 5 cm above the center of the load axle; road-wheel diameter (d_w) was 17.8 cm in all tests; slip (S) was held constant at 20 percent; drive sprocket pitch radius (r) was 16.51 cm for all tests; and no linear dimensions (ℓ''s) were considered that are not specified in the first 21 variables listed in Table 1. Of the 16 remaining variables, four were found to influence P_{20} most: G, b, ℓ, and W . A subsequent dimensional analysis showed that the dimensionless functional relation among P_{20}, G, b, ℓ, and W can be expressed as $P_{20}/W = f(b/\ell, G\ell^3/W)$. In the following paragraphs, dimensional analysis is used to obtain a much more comprehensive set of dimensionless Pi terms for the track-soil system than was produced in Report 2. The Pi terms developed serve as a base for describing track performance in either frictional or cohesive soils. Functional equations were developed only for the track- (air-dry) sand system.

Independent and Dependent Variables Considered

Independent variables

19. The independent variables of the soil-single track system can be divided into three groups: soil variables, single-track variables, and system variables.

Variable	Symbol	Mass, Length, Time (MLT) Units
Soil:		
Friction angle	ϕ	-
Cohesion	c	$ML^{-1}T^{-2}$
Density	γ	$ML^{-2}T^{-2}$
Spissitude	η	$ML^{-1}T^{-1}$
Single-Track:		
Width	b	L
Contact length	ℓ	L
Angle of approach	α	-*
Angle of departure	β	-
Horizontal location of at-rest center of gravity	RCG_h	L
Vertical location of at-rest center of gravity	RCG_v	L
Road-wheel spacing	s_w	L
Road-wheel diameter	d_w	L
Minimum pressure in road bogies	p_b	$ML^{-1}T^{-2}$
Distribution of pressure in road bogies	dp_b	$f(ML^{-1}T^{-2})$
Drive-sprocket location	ds	-**
Track-shoe height	h_s	L
Track-shoe thickness	th_s	L
Track-shoe spacing	s_s	L

* α and β each may be described as an angle, or as the ratio of vertical-to-horizontal distances that define the angle.

** "Front" and "rear" are adequate descriptions of drive sprocket location for the WES test rig, in which α and β are controllable independently.

Variable	Symbol	Mass, Length, Time (MLT) Units
Index of track-belt tension	t_{tb}	$ML^{-1}T^{-2}$
Drive sprocket pitch radius	r	L
Other pertinent track dimensions	ℓ'	L
System:		
Load	W	MLT^{-2}
Actual translational velocity	V_a	LT^{-1}
Slip	S	-
Track-soil friction	f	-
Acceleration due to gravity	g	LT^{-2}

Dependent variables

20. The dependent variables of the soil-track system are the major performance characteristics:

Variable	Symbol	Mass, Length, Time (MLT) Units
Pull (often with percent slip as a subscript)	P_{20}	MLT^{-2}
Towed force	P_T	MLT^{-2}
Torque (often with percent slip as a subscript	M_{20}	ML^2T^{-2}
Sinkage (often with location relative to track as a subscript)	z_R	L
Trim angle*	θ	-

The Pi Terms

21. Dimensional analysis is a technique based on a consideration of the variables that describe a system, and a requirement that

* In fixed-trim-angle tests, θ becomes an independent variable, and one or more new dependent variables are required (within the framework of this analysis) to account for the manner in which such nominally independent variables as load and at-rest center of gravity are changed by the test mechanism used to restrain θ (see paragraph 47).

expressions relating these variables must be dimensionally balanced. Through use of this technique, the investigator can obtain partial information about the interrelations among the variables associated with a particular phenomenon. Several investigators have critcially examined the assumptions and limitations of dimensional analysis and described the mechanics of applying it.[6-8]

22. In the present application, each of the five dependent variables of interest has been postulated as a function of 26 independent variables involving three dimensions, mass, length, and time. According to dimensional theory, removing all three dimensions from this system will produce five dependent dimensionless variables (dependent Pi terms) as functions of 23* independent dimensionless variables (independent Pi terms). These Pi terms may be formed by inspection or by any of several more formal procedures.[6-8] The set of Pi terms produced by any procedure can be subsequently manipulated by the analyst to incorporate his special knowledge and insights, and/or to accommodate his test program to prediction needs. The final set will be as valid as the initial set, provided that it contains the same number of Pi terms and that these are still all independent within the set.

23. One such initial set of 23 Pi terms to describe the independent variables of the single track-soil system, formed by assigning contact length (ℓ), load (W), and actual translational velocity (V_a) as the most basic system parameters, is as follows:

a. Soil variables (4)

$$\pi_1 = \phi \qquad \pi_3 = \frac{\gamma \ell^3}{W}$$

$$\pi_2 = \frac{c \ell^2}{W} \qquad \pi_4 = \frac{V_a \eta \ell}{W}$$

b. Single-track variables (16)

$$\pi_5 = \frac{b}{\ell} \qquad \pi_7 = \beta$$

$$\pi_6 = \alpha \qquad \pi_8 = \frac{RCG_h}{\ell}$$

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* Twenty-six variables minus three dimensions.

$$\pi_9 = \frac{RCG_v}{\ell}$$

$$\pi_{10} = \frac{s_w}{\ell}$$

$$\pi_{11} = \frac{d_w}{\ell}$$

$$\pi_{12} = \frac{p_b \ell^2}{W}$$

$$\pi_{13} = \frac{dp_b \ell^2}{W}$$

$$\pi_{14} = ds$$

$$\pi_{15} = \frac{h_s}{\ell}$$

$$\pi_{16} = \frac{th_s}{\ell}$$

$$\pi_{17} = \frac{s_s}{\ell}$$

$$\pi_{18} = \frac{t_{tb} \ell^2}{W}$$

$$\pi_{19} = \frac{r}{\ell}$$

$$\pi_{20} = \frac{\ell}{\ell}$$

c. System variables (3)

$$\pi_{21} = S \text{ or } \left(1 - \frac{V_a}{V_t}\right)$$

$$\pi_{22} = f$$

$$\pi_{23} = \frac{g\ell}{V_a^2}$$

24. Five suitable Pi terms for the selected dependent variables are:

$$\pi_{24} = \frac{P}{W}$$

$$\pi_{25} = \frac{P_T}{W}$$

$$\pi_{26} = \frac{M}{\ell W} \cdot \frac{1}{\pi_{19}} = \frac{M}{r\omega}$$

$$\pi_{27} = \frac{z}{\ell}$$

$$\pi_{28} = \theta^*$$

25. The 23 independent Pi terms in paragraph 23 provide a comprehensive base upon which can be developed a detailed description of straight-line track performance in level soil as defined by the five dependent Pi terms in paragraph 24. Note that the variables considered to describe soil included terms capable of describing soils that derive their strength either entirely from friction (ϕ soils), from both friction and cohesion (c-ϕ soils), or from cohesion only (c soils). Thus, the 28 Pi terms can be used to develop a description of track performance in practically any soil.

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* See footnote, page 19.

Selection of Pi Terms for the Sand-Track System

26. For any particular segment of the soil-track system, the number of Pi terms needed for an adequate description of track performance is usually much less than the full listing of 28 in paragraphs 23 and 24. Determination of the Pi terms to use is based primarily on two considerations: (a) pertinence of the independent and dependent variables to the system under investigation, and (b) the amount of detail sought in the description of the system.

Pertinence to the system of concern

27. Soil variables. Because the strengths of both test soils in this study (Yuma and mortar sands) derive almost entirely from friction, only soil variables friction angle (ϕ) and density (γ) from paragraph 19 need to be considered. Further simplification results from two related observations. First, several experimenters have shown that friction angle of a given cohesionless dry sand is proportional to its density.[3,9] Also, it has been determined that penetration resistance gradient (G) is a sensitive indicator of density change in a frictional soil (Reference 3 and Figure 4). Thus, G is indicated sufficient to substitute for variables ϕ and γ, and is the only soil variable included in subsequent analyses of this report. G has units $ML^{-2}T^{-2}$; accordingly, the Pi term

$$\pi_{29} = \frac{G\ell^3}{W}$$

can be used in place of π_1, π_2, and π_3. In Report 2 of this series,[2] from analysis of data developed to that point, Pi term π_{29} was further developed to basic-variable prediction term

$$\pi_{30} = \frac{G(b\ell)^{3/2}}{W} = N_{BV} \text{ (equation 1)}$$

which is dimensionally equivalent and conveys more information. This form of the term will be used as the basis of the development that follows.

28. Single-track variables. Only one road-wheel diameter was used in the tests reported herein, so variable d_w was omitted. Also, in the test system, drive-sprocket pitch radius (r) was constant at

16.51 cm and appears only in connection with sprocket torque, as reflected in the formation of π_{26}. Other minor linear dimensions (ℓ's) were considered negligable. Thus π_{11}, π_{19}, and π_{20} need not be carried further in this study.

29. System variables. Data from most tests reported herein were examined at a constant or near-constant value of slip, so S was eliminated as an active variable. Also, soil-to-soil rather than track-to-soil failures were observed in all tests reported herein; thus, the track-soil friction term (f) was deleted. Finally, track translational velocities in the test series were so low (0.6 m/sec) that system forces deriving from soil inertia were negligable. This, in effect, removes the Pi term π_{23} in which the acceleration due to gravity enters explicitly as a system variable. Thus, none of the Pi terms describing the system variables will be considered further.

30. Final Pi terms of concern. The considerations in paragraphs 27-29 reduce the number of independent Pi terms of concern from 23 (listed in paragraph 23) to 15.*

Pi Term		Descriptive Title
π_{30}	$G(b\ell)^{3/2}/W$	Sand loading number
π_4	$V_a \eta \ell / W$	Velocity number
π_5	b/ℓ	Track shape number
π_6	α	Approach number
π_7	β	Departure number
π_8	RCG_h/ℓ	RCG number, horizonal
π_9	RCG_v/ℓ	RCG number, vertical
π_{10}	s_w/ℓ	Road-wheel spacing number
π_{12}	$p_b \ell^2/W$	Bogie pressure number
π_{13}	$dp_b \ell^2/W$	Bogie pressure distribution number
π_{14}	ds	Sprocket location designator
π_{15}	h_s/ℓ	Track-shoe height number
π_{16}	th_s/ℓ	Track-shoe thickness number
π_{17}	s_s/ℓ	Track-shoe spacing number
π_{18}	$t_{tb} \ell^2/W$	Track tension number

* Additionally, π_{28}, θ, is an independent Pi term in fixed-angle tests. See footnote, page 19.

31. Each of the five dependent variables listed in paragraph 20 is considered herein. The Pi terms associated with these variables are:

Pi Term	Descriptive Title	Pi Term	Descriptive Title
$\pi_{24} = P/W$	Pull coefficient	$\pi_{27} = z/\ell$	Sinkage coefficient
$\pi_{25} = P_T/W$	Towed-force coefficient	$\pi_{28} = \theta$	Trim angle number
$\pi_{26} = M/Wr$	Torque coefficient		

Detail of description

32. The number of Pi terms in paragraphs 30 and 31 indicates the complexity of the sand-track system under examination. Fortunately, a useful description of this system does not need to include functions of all of the 15 independent Pi terms in paragraph 30. In Report 2, the effects on P_{20}, pull at 20 percent slip, of 16 independent variables were investigated in three Plackett-Burman (statistical) screening test designs. The primary intent of those designs was to determine a ranking of importance of the variables examined relative to their effects on P_{20}. The Plackett-Burman design is well suited to this task since it allows the effect of a given variable on the test response to be determined independent of the influence of the remaining variables whose effects are being studied in the design.[2,10]

33. The effect of a given variable (load W, for instance) on the test response (P_{20} in this case) in a Plackett-Burman design is computed as

$$\text{Effect of W} = E_W = \frac{\text{Sum of } P_{20}\text{'s with W } (+)}{n/2} - \frac{\text{Sum of } P_{20}\text{'s with W } (-)}{n/2}$$

where n = number of tests in the Plackett-Burman design of which W is one of the variables with low and high values [i.e. (-) and (+) values].* Note that a negative effect indicates that changing the value of the variable from its low to its high level causes the value of the test response to decrease.

* The (-) and (+) signs used to describe the low- and high-level values, respectively, of independent variables in the Plackett-Burman designs, have no algebraic meaning; they serve only to indicate a low- or high-level test value. Report 2 of this series gives more details relative to Plackett-Burman designs in general, and to the manner in which such designs were applied to the sand-model track system in particular.

34. One means of expressing the results obtained in the three Plackett-Burman designs in Report 2 is by the percent effect factor for pull, (effect on P_{20}/ average value of P_{20}) × 100, which indicates the change in percent from the average value of P_{20} caused by testing a variable at its low and high values. Values of this term for the three designs follow:

Plackett-Burman Design No. 1

Variable	Percent Effect Factor for Pull
G	19.8
W	69.7
b	37.5
ℓ	17.6

Average value of P_{20} = 2616 N

Plackett-Burman Design No. 2

Variable	Percent Effect Factor for Pull
α	- 6.3
β	0.5
V_a	4.4
t_{tb}	6.6
s_s	- 0.2
th_s	- 1.0
p_b	-18.2
dp_b	- 2.7

Average value of P_{20} = 2111 N

Plackett-Burman Design No. 3

Variable	Percent Effect Factor for Pull
h_s	5.9
s_w	-37.0
ds	6.9
θ	-94.6

Average value of P_{20} = 2368 N

In the tabulation above, the importance of a given variable relative to its effect on P_{20} increases as its absolute (i.e. nonalgebraic) value increases.

35. Variables G, W, b, and ℓ are hereafter called the "basic" sand-track variables since they are included in the basic-variable prediction term (equation 1) developed in Report 2 to describe track pull coefficient P_{20}/W. Variables p_b, s_w, and θ (or alternatively, RCG_h and RCG_v for tests where θ is a dependent variable) are termed the "secondary" track variables because their absolute tabulated values

in paragraph 34 are large enough to suggest relatively strong influence on P_{20}. Track variables α, β, V_a, t_{tb}, s_s, th_s, dp_b, h_s, and ds will be referred to as the "other" track variables because their tabulated values in paragraph 34 are relatively small.

Functional Equations for the Sand-Track System

36. The overall frame of reference used herein to describe single-track performance in air-dry sand is provided by the following general equations (using Pi terms from paragraphs 30 and 31):

$$\frac{P}{W} = f^1\left(\frac{G(b\ell)^{3/2}}{W}, \frac{V_a \eta \ell}{W}, \frac{b}{\ell}, \alpha, \beta, \frac{RCG_h}{\ell}, \frac{RCG_v}{\ell}, \frac{s_w}{\ell}, \frac{p_b \ell^2}{W}, \frac{dp_b \ell^2}{W}, ds, \frac{h_s}{\ell}, \frac{th_s}{\ell}, \frac{s_s}{\ell}, \frac{t_{tb} \ell^2}{W}\right)$$

$$\frac{P_T}{W} = f^2\left(\qquad \text{"} \qquad \right)$$

$$\frac{M}{Wr} = f^3\left(\qquad \text{"} \qquad \right)$$

$$\frac{z}{\ell} = f^4\left(\qquad \text{"} \qquad \right)$$

$$\theta = f^5\left(\qquad \text{"} \qquad \right)$$

37. The influence of the secondary and some of the other track variables (paragraph 35) on pull coefficient will be analyzed in the following part of this report to determine (a) whether the relation of the basic-variable prediction term to pull needs to be modified to account for the influence of these variables on the pull coefficient, and (b) if so, what form this modification should take. In all cases where the prediction term is modified, the modifier will be in a form related to one of the dimensionless Pi terms in parentheses in paragraph 36.

$$\frac{P}{W} = f^1\left(\frac{G(b\ell)^{3/2}}{W}, \frac{V_a \eta}{W}\right.$$

$$\frac{P_T}{W} = f^2\left(\right.$$

$$\frac{M}{Wr} = f^3\left(\right.$$

$$\frac{z}{\ell} = f^4\left(\right.$$

$$\theta = f^5\left(\right.$$

PART IV: ANALYSIS OF DATA

Influence of Secondary and Other Track Variables on the Relation of Pull Coefficient to the Mobility Number

Basic-variable prediction term

38. The basis for the analysis in this report is the basic-variable prediction term (equation 1) developed in Report 2. The least-squares linear relation of best fit between the pull coefficient and log of the basic-variable prediction term established in Report 2 is shown in Plate 1. Data in Plate 1 include values of G from 0.90 to 6.82 MN/m^3 (in Yuma sand); W from 1,291 to 25,162 N; b values of 15.2, 30.5, and 61.0 cm; and ℓ values of 61.0 and 121.9 cm. The remaining independent variables considered in paragraph 30 were held constant in the tests plotted in Plate 1 at the following values:

Pi Term	Major Independent Variable	Constant Value in Plate 1
π_4	V_a	0.6 m/sec
π_6	α	22 deg
π_7	β	22 deg
π_8	RCG_h	At horizontal center line
π_9	RCG_v	Center of load axle to 4 cm above center
π_{10}	s_w	20.3 cm (all road wheels in)
π_{12}	p_b	276 kPa
π_{13}	dp_b	Uniform
π_{14}	ds	Rear
π_{15}	h_s	2.5 cm
π_{16}	th_s	0.32 cm
π_{17}	s_s	3.0 cm
π_{18}	t_{tb}	6890 kPa

Influence of secondary track variables on pull coefficient

39. Pressure in road bogie cylinders (p_b). The support element of each road bogie of the WES model track is a pneumatic cylinder,[1]

and maximum bogie deflection (10 cm) is reached when the bogie is loaded to 8.1 N per kilopascal of bogie-cylinder pressure. To determine how pressure in the road-bogie cylinders affects pull coefficient, tests were conducted with the model track in air-dry mortar sand at three levels of pressure: 276, 448, and 621 kPa. Pretest values of the 12 variables other than pressure (p_b) listed in paragraph 38 were set at the values shown there. Test results are listed in Table 2. For these tests, Plate 2a shows a well defined relation between pull coefficient and log of the basic-variable prediction term. There is a definite separation of the data by pressure level, however, with values of pull coefficient at a given value of the prediction term generally increasing as values of pressure decrease (i.e., as the ability of the road bogies to resist deflection decreases).

40. Collapse of the data in Plate 2a could be achieved using some function of pressure. It was judged more meaningful, however, to use a function of some variable such as road-bogie deflection (Δ),* which is common to all types of prototype tracked vehicle suspensions. A dimensionless term descriptive of track suspension that includes deflection is the deflection ratio Δ/Δ_{max}, where Δ_{max} is the maximum deflection before the bogie "bottoms out." For the model track, bogie deflection changes in near-linear fashion with load below that load (W_{max}) that causes maximum deflection. Since maximum deflection for each bogie is reached at 8.1 N per kilopascal of bogie-cylinder pressure, W_{max} (in newtons) for the overall model track is computed as p_b × number of bogies × 8.1. The upper limit of the weight ratio W/W_{max} is taken as 1.00 because load (W) greater than W_{max} cannot cause bogie deflection greater than maximum. The weight ratio W/W_{max} is a reasonable nominal estimate of the deflection ratio (Δ/Δ_{max}), and is used hereafter to characterize track suspension.

41. Values of the weight ratio for the data in Plate 2a ranged from 0.29 to 1.00, 0.19 to 1.00, and 0.09 to 1.00 for bogie-cylinder

* Δ was measured while load was applied vertically to the track as it rested on a flat, level unyielding surface.

pressures 276, 448, and 621 kPa, respectively. Analysis of the arrangement of these values led to the use of the dimensionless multiplicative term $(W/W_{max})^{1/2}$ with the basic-variable prediction term to account for the influence of suspension on pull coefficient (Plate 2b). Significantly less data scatter is present in Plate 2b than in Plate 2a (standard error of estimate values of 0.032 and 0.041, respectively), demonstrating that the multiplicative term is useful in describing the influence of track suspension on in-sand track pull coefficient.*

42. The log-linear relation in Plate 2b is well defined from test results in mortar sand, but is based on data none of whose abscissa term values are larger than 50. To extend the relation in Plate 2b to much larger values of this term, the same Yuma sand test data used in Plate 1 are plotted in Plate 3. The solid and dashed lines used to describe the relation in Plate 3 (Yuma sand data) are the same as those used in Plate 2b (mortar sand data) for abscissa values up to 25. For values larger than 25, the Yuma sand data in Plate 3 indicate a progressive decrease in the slope of the curve, with an upper limit of the pull coefficient of about 0.6 indicated for values of the abscissa term beyond about 500.

43. Vertical separation between the dashed lines in Plate 3 is constant at 0.128. These lines specify boundaries that include most of the Yuma sand data (Plate 3), as well as the coordinates of nearly all the mortar sand data (Plate 2b). This indicates that penetration resistance gradient G is effective in characterizing the strength of these two markedly different sands on a common basis insofar as track pull coefficient is concerned. The curve and error band in Plate 3 are used as the basis of comparison in some of the subsequent analyses and are hereafter referred to as the "standard" curves.

44. Road-wheel spacing (s_w). For the model track, the road-wheel diameter was 17.8 cm for all tests and the spacing between center lines

* The value $s = 0.032$ from the relation in Plate 2b compares favorably with $s = 0.04$ established for the pull coefficient versus dimensionless prediction term relation established in WES laboratory tests of pneumatic tires in air-dry desert sand.[11]

of adjacent road wheels was 20.3 cm for all routine tests. Eighteen special tests were conducted in Yuma sand at the two other model-track road-wheel spacing values--nine at 40.6 cm and nine at 61.0 cm. Pretest values of all other variables listed in paragraph 38 were set at the values shown, except for road-bogie cylinder pressure, which was set at 621 kPa. Test results are presented in Table 3, and the relation of pull coefficient versus log of $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$ in Plate 4. Compared to the "standard" curves superimposed from Plate 3 onto Plate 4, the test data in Plate 4 developed, on average, values of pull coefficient about 0.06 smaller than indicated by the curves. Not surprisingly, data for the 61.0-cm spacing show considerably more scatter in Plate 4 than do those for 40.6-cm spacing, but data for both spacings generally parallel the pattern of the "standard" curves.

45. A reasonable system variable to use for linear term ℓ in $\pi_{10} = s_w/\ell$ (paragraph 30) is road-wheel diameter (d_w) so that a spacing-diameter ratio (s_w/d_w) of 1.0 indicates adjacent wheels are touching, and values of s_w/d_w larger than 1.0 indicate how many diameters minus one apart are the center lines of adjacent wheels. The spacing-diameter ratio for prototype tracked vehicles usually takes fairly small values (in the order of 1.1 to 1.3) because of considerations of excessive weight concentration and because vehicle ride can be expected to become more harsh as spacing between road wheels increases. The value of the ratio in Plate 3 is within this range (at 1.14), while those in Plate 4 are much larger (at 2.3 and 3.4). Thus, the relation in Plate 3 is judged adequate to describe in-sand track pull performance for values of the spacing-diameter ratio ordinarily encountered in prototype tracked vehicles.

46. Trim angle (θ) and at-rest center of gravity (RCG). In the Plackett-Burman tests described in Report 2 and referred to in paragraphs 32-35 herein, track-frame trim angle (θ) was mechanically maintained at a preselected value in each test. In real-world applications, the trim angle generally is controlled in the design of a vehicle only insofar as the location of its at-rest center of gravity (RCG) is concerned. It was considered reasonable, then, to study the effect on

track performance caused by moving the at-rest center of gravity forward or rearward of the model track's geometric center line and not artificially maintaining a given trim angle.

47. Tests were conducted with the 30.5- by 61.0-cm track with the load axle located either 15.2 cm forward or rearward of the track's geometric center line, and with the 30.5- by 121.9-cm track with the load axle either 45.7 cm forward or rearward of the center line. Test results are presented in Table 4. All components of the model track were weighed and the locations of their individual centers of gravity measured relative to a fixed reference point. Load applied at the load axle was treated as a track "component" whose weight was determined by subtracting from overall test load the sum of weights of the physical components of the track. For the track resting on a flat, level, unyielding surface, overall track weight plus load was computed to act vertically at a position d (cm) forward of the center line of the rearmost road wheel, where

$$d = \sum_{\substack{\text{all}\\ \text{components}}} \left(\begin{matrix}\text{component}\\ \text{weight}\end{matrix} \times \begin{matrix}\text{horizontal distance from rear road}\\ \text{wheel to center of gravity of component}\end{matrix} \right) \div \sum_{\substack{\text{all}\\ \text{components}}} (\text{weight} + \text{load})$$

Thus, for the same track and the same position of the load axle, different values of d were obtained, depending on the magnitude of load applied at the load axle. The term d designates the location of the effective horizontal at-rest center of gravity (RCG_h) of the unrestrained track with pull acting parallel to the sand surface and through the load axle. No study was made of the influence of the vertical at-rest center of gravity (RCG_v) on model track performance because its location varied only from 0 to about 4 cm above the load axle for the full range of load conditions possible with the model track.*

* The magnitude of the test load had little influence on the position of the RCG_v. This results because, with zero load applied at the load axle, the RCG_v of both the short and long model track configurations is only about 4 cm above the axle. Increasing load at the axle causes the RCG_v to approach the same height as the load axle. For the analysis herein, the location of the RCG_v above the track base is assumed constant at 0.55 times overall model track height.

48. Plate 5a shows the relation of pull coefficient to the log of $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$ for tests in Yuma sand with the at-rest center of gravity either forward of the geometric center line (open data points) or rearward of the center line (closed points). The number beside each data point is the value of the horizontal distance from the center line of the track rear road wheel to the at-rest center of gravity divided by half the track contact length [$(d/(\ell/2)$]. This term is a useful dimensionless indicator of the location of the at-rest center of gravity relative to the size of the track, as will be seen.

49. The solid and dashed curves in Plate 5a occupy the same positions as those in Plate 3, where all the data considered had the at-rest center of gravity at the track geometric center line [$d/(\ell/2) = 1.0$]. The form of the abscissa term in Plates 3 and 5a can be altered by multiplicative terms that include $d/(\ell/2)$ to cause the curve from Plate 3 to describe the pull performance of tracks with RCG_h either rearward, forward, or at the geometric center line. For example, the closed-symbol data points (RCG_h rearward of center line) can be moved leftward to cluster about the solid curve by multiplying the abscissa term by $\left(\frac{d}{\ell/2}\right)^{3/2}$ (see Plate 5b).

50. All six of the open-symbol data in Plate 5a lie inside the dashed lines, indicating that, at most, very little account needs to be taken for the at-rest center of gravity forward of the geometric center line [for $d/(\ell/2)$ values up to 1.55]. It may by significant that values of pull coefficient somewhat larger than indicated by the solid curve in Plate 5a were obtained for the three open-symbol data points with the smallest abscissa values. (The open-symbol data point with the next larger abscissa value lies nearly atop the solid curve, and the two other open-symbol data points have abscissa values within the range where changes in pull coefficient values are relatively insensitive to changes in abscissa-term values.) Locating the at-rest center of gravity forward of the center line decreases the natural tendency of the track to assume its tail-down attitude at 20 percent slip, thereby promoting a smaller track trim angle. For the heavy track and/or

low-strength sand condition (i.e. for small values of the abscissa term in Plate 5a), it is difficult to imagine that this could but improve track performance. That is, other conditions being equal, a track with RCG_h forward of center line should have a prediction term (abscissa) larger than one whose RCG_h is at the center line. Accordingly, the open-symbol data in Plate 5a have been shifted slightly to the right by multiplying the abscissa term by $\left(\frac{d}{\ell/2}\right)^{1/2}$ and are shown in Plate 5b.

51. In summary, the "standard" curves from Plate 3 can be used for tracks with the at-rest center of gravity located rearward of the center line by changing the predictive abscissa term from Plate 3 to the sand-track mobility number, N_s, expressed as

$$N_s = \frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2} \cdot \left(\frac{d}{\ell/2}\right)^{3/2} \tag{2}$$

and for tracks with the at-rest center of gravity forward of center line by using

$$N_s = \frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2} \cdot \left(\frac{d}{\ell/2}\right)^{1/2} \tag{3}$$

Note that for tracks with at-rest center of gravity at the center line, $d/(\ell/2) = 1.0$, so that both $\left(\frac{d}{\ell/2}\right)^{3/2} = 1.0$ and $\left(\frac{d}{\ell/2}\right)^{1/2} = 1.0$. The general form of the sand-track mobility number can be expressed as

$$N_s = \frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2} \cdot \left(\frac{d}{\ell/2}\right)^{n} \tag{4}$$

where $n = 1$, $3/2$, or $1/2$ depending on whether the track at-rest center of gravity is at, rearward of, or forward of, respectively, the geometric center line.

Influence of other track variables on pull coefficient

52. <u>Elimination of some variables</u>. The other variables among α, β, V_a, t_{tb}, s_s, th_s, dp_b, h_s, and ds (paragraph 35) that merited further investigation were selected primarily from considerations of (a) the range of test conditions possible with the WES model track equipment, (b) the range of conditions included in the Plackett-Burman tests (from Report 2), and (c) observations of the behavior of the other

variables during tests to study the effects of the basic and secondary variables.

53. In considering angle of approach (α), it was noted in virtually all tests at 20 percent slip that the model track assumed a tail-down attitude with very little, if any, of the track that lies in front of the front road bogie in contact with the sand. This, together with the fact that the low-level (-)and high-level (+) values of α in the Plackett-Burman tests were fairly widely separated (22- and 30-deg with all bogies fully extended, from Report 2) and set at nearly the extreme values possible with the model track (paragraph 8) led to the conclusion that no further study needed to be made with the model track on the influence of angle of approach on pull coefficient.*

54. Low and high levels of departure angle (β) were also set at 22 and 30 deg in the Plackett-Burman tests, and a very small value (0.5) was obtained for the percent effect factor for pull (paragraph 34). No further study of track departure angle was indicated necessary.

55. Actual track translational velocity (V_a) was omitted from further consideration since its percent effect value for pull was quite small (4.4), and because the maximum velocity possible with the model track is only 0.6 m/sec.

56. An extremely small value (-0.2) was obtained for the percent effect value for pull caused by track-shoe spacing (s_s) in the Plackett-Burman tests, even though the low and high levels of spacing were widely different--3.0 and 14.2 cm, respectively. For operation in air-dry sand, track pull appears, then, to be related primarily to sand bearing strength and to be affected very little by the spacing between adjacent track shoes. Variable s_s was eliminated from further study.

57. Tests beyond the Plackett-Burman test series were not conducted to determine the effects of changing track-shoe thickness (th_s) since a very small value of the percent effect factor for pull, 1.0, was obtained for this variable (paragraph 34).

58. Except for four Plackett-Burman tests, only one track-shoe

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* For nearly all prototype tracked vehicles, the value of α is about 20 deg or larger.

height (h_s), 2.5 cm, was used in all of the model track tests. This was done because (a) a relatively small value (5.9) was obtained for the percent effect factor for pull, and (b) the 5.1-cm-high track shoes used in the four Plackett-Burman tests which included high-level height values, were found unable to withstand the bending forces exerted on the shoes by the sand during testing.

59. Overall, then, of the nine other track variables listed in paragraph 52, considerations in paragraphs 53-58 eliminate all but three for further study--t_{tb}, dp_b, and ds.

60. <u>Index of track-belt tension (t_{tb})</u>. A detailed description of the physical significance of the measurement index of track-belt tension (t_{tb}) was included in Report 1. Briefly, this measurement indicates the relative tightness of the rubber-and-fabric track belt in the vicinity of the idler sprocket, measured in terms of the outward pressure (and force) exerted on the idler sprocket by a constantly monitored hydraulic jack (Figure 7). The free-body diagram in Figure 7 was solved in

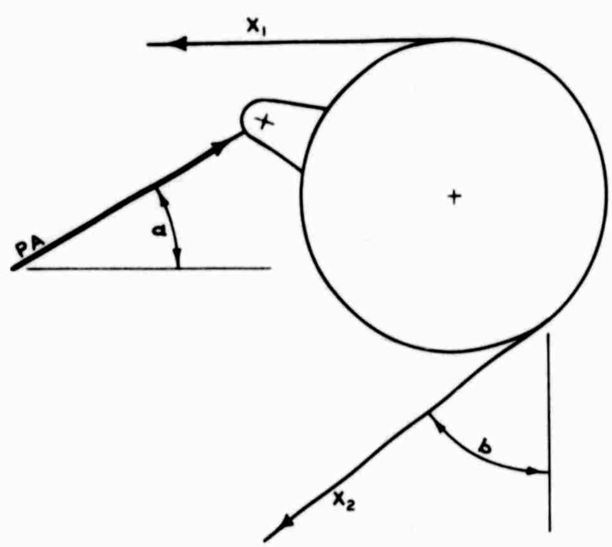

Figure 7. Force at track idler sprocket

Report 1 to show that X_1 = PA cos a - (sin a) (tan b) and X_2 = PA $\frac{\sin a}{\cos b}$ * (where P = hydraulic jack pressure and A = jack inner-chamber cross-sectional area, 7.9 cm^2). Suffice it to say that for tension index = 1380 kPa, the track belt was relatively slack; for tension index = 4140 kPa, it was moderately taut; and for tension index = 6890 kPa, it was very taut.

61. From paragraph 34, the value of percent effect factor for pull for track-belt tension index is 6.6. To define the influence of tension index on pull coefficient in more detail, tests were conducted in Yuma sand with four track geometries and all variables in paragraph 38, except road-bogie cylinder pressure p_b and track tension index t_{tb}, set at the values shown there. The pretest value of bogie pressure was 621 kPa for all tests, and that of track-belt tension index was 1380 kPa for half of the tests and 4140 kPa for the other half. Tests results are listed in Table 5, and the relation of pull coefficient to log of the sand-track mobility number is shown in Plate 6.

62. In Plate 6, the standard curves (Plate 3), where tension index = 6890 kPa, are used as the basis for comparison. On average, the data for a track tension of 4140 kPa (closed symbols) lie about 0.02 below the "standard" curves, and those of 1380 kPa (open symbols) about 0.06 below. Both the closed- and the open-symbol data generally parallel the pattern of the "standard" curves. A decrease in pull coefficient of only about 0.06 caused by lowering the index of track-belt tension rather drastically (from 6890 to 1380 kPa, or from very taut to relatively slack) indicates that track-belt tension affects track pull only slightly. The worsening of pull performance that accompanies a reduction in track-belt tension likely results from overall track load becoming concentrated more and more at the road wheels as belt tension decreases--i.e. lowering tension lowers the ability of the track belt itself to support vertical load, and so reduces the ability of the track to distribute load uniformly along its

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* Angle α was approximately 25 deg and angle β 55 deg for most track conditions tested.

ground contact length. It is recommended that tracks be operated as taut as practicable, in which case the relation in Plate 3 would apply. For moderately taut-to-slack tracks, pull coefficient performance can be expected to decrease from that described by the "standard" curves by as much as 0.06 or so.

63. Distribution of pressure in road-bogie cylinders (dp_b). The value of the percent effect factor for pull obtained in the Plackett-Burman tests for distribution of pressure in the bogie cylinders (dp_b) was quite small, -2.7. In these tests, however, the difference in pressure between adjacent road-bogie cylinders was only 20.7 kPa, so that even for the long track configuration, the difference in values of pressure between the two end road-bogie cylinders was only 145 kPa. Tests were conducted later with both long (121.9 cm) and short (61.0 cm) track configurations, in which the pretest bogie-cylinder pressures were set at levels that either increased or decreased linearly from the front to the rear of the track. In each of these tests, values of road-bogie cylinder pressure (p_b) varied from 103 to 621 kPa from one end of the track to the other. Results of these tests are listed in Table 6, and the associated relation of pull coefficient versus log of the basic-variable prediction term is shown in Plate 7a.

64. Although the relation in Plate 7a is well defined, the data appear to separate by distribution of bogie-cylinder pressure, with data for the decreasing front-to-rear pattern (closed-symbol data) generally developing larger values of pull coefficient at common values of the basic-variable prediction term. From paragraph 39, it was learned that values of pull coefficient at a given value of the prediction term generally increase as values of bogie pressure decrease. It has been noted several times, too, that the model track almost always assumed a tail-down attitude at 20 percent slip. Thus, it should be expected that track pull at 20 percent slip is affected more by conditions at the rear, rather than at the front end of the track.

65. Taking the above observations into account, the abscissa term in Plate 7a was changed to the sand-track mobility number (equation 5) by computing W_{max} on the basis of pressure in the track's

rear road-bogie cylinder only (Plate 7b). The success of this operation is illustrated by the substantial reduction in data scatter in Plate 7b versus that in Plate 7a. It is significant, too, that the relation in Plate 7b is described quite well by the "standard" curves (Plate 3), indicating that the abscissa term in Plate 7b is computed on a basis compatible with that in Plate 3.

66. Drive sprocket location (ds). The value of the percent effect factor for pull for drive sprocket location (paragraph 34) was fairly substantial, 6.9. To study the effect of this variable in some detail, tests were conducted in Yuma sand with all six track geometries and the drive sprocket at the front of the track in each test; test results are presented in Table 7. All of the other variables listed in paragraph 38 were set at the values shown there, except that bogie pressure (p_b) was 621 kPa in each test. Results of these tests are shown in Plate 8 in the pull coefficient versus log of the sand-track mobility number relation. Compared to the "standard" curves developed in Plate 3 using data from rear-sprocket-drive tests and transferred from there to Plate 8, the test data in Plate 8 generally parallel these curves but have ordinate values that are, on average, about 0.04 smaller. Very likely this result derives from a somewhat less favorable distribution of ground pressure associated with the front-sprocket-drive test condition.

67. Degrading track performance by only 0.04 seems little price to pay for choosing front- rather than rear-sprocket drive. Still, the slight difference between pull coefficient performance for front- and rear-sprocket drive is considered to be real, and rear-sprocket drive is recommended.*

68. Summary. The relation in Plate 3 is considered adequate to predict pull coefficient under the influence of the full range of values tested for all variables considered herein, except for location of the at-rest center of gravity (RCG), index of track-belt tension (t_{tb}), and drive-sprocket location (ds). For the at-rest center of

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* Most recent prototype tracked vehicles are in fact rear-sprocket driven.

gravity located forward or rearward of the track geometric center line, the same relation as in Plate 3 is used, but the dimensionless prediction term in Plate 3 is expanded to the sand-track mobility number (Equation 4), as shown in Plate 5b. The "standard" curves in Plate 3 are applicable for taut tracks; moderately taut-to-slack tracks produce values of pull coefficient smaller by about 0.06 than those of the standard curves (paragraph 62 and Plate 6b). Locating the drive sprocket at the front of the track causes values of pull coefficient to be smaller than those of the standard curves by about 0.04 (Plate 8). Increasing road-wheel spacing degrades track pull coefficient performance (Plate 4), but the relation in Plate 3 adequately describes track pull coefficient for the road-wheel spacing-to-diameter ratios ordinarily encountered in prototype vehicles (s_w/d_w values from about 1.1 to 1.3)--see paragraphs 44 and 45.

Prediction of Performance Terms Other Than Pull Coefficient

Relations involving torque, sinkage, and trim angle at 20 percent slip

69. Torque. For the model track, values of net torque that are essentially free of the influence of internal motion resistance are used exclusively herein. Net torque (M) equals $M_G - M_I$, where M_G is gross torque measured during the in-soil test, and M_I is the pretest torque required to rotate the track in air at the same velocity used in the subsequent in-soil test.

70. The relation of torque coefficient* at 20 percent slip (M_{20}/Wr) to log of the sand-track mobility number is shown in Plate 9. Data for all of the tests in Tables 2 through 7 are shown in this plate, so that two sand types are represented, along with a wide range of values for each of variables, G, W, b, ℓ, p_b, s_w, RCG_h, dp_b, ds, and t_{tb}. In Plate 9, nearly all values of torque coefficient lie within a range from 0.4 to 0.9, and are insensitive to changes in the value of the sand-track mobility number.

* r in torque coefficient M_{20}/Wr is track drive-sprocket pitch radius, 0.1651 m for all tests in this report.

71. The reason for the lack of association among the variables in Plate 9 can be determined from Plate 10a. Here, data from the same tests as in Plate 9 show that torque at 20 percent slip increases linearly with the product of load times drive-sprocket radius (Wr) and has little dependence on track size or shape. Values of soil penetration resistance gradient (G) from 0.69 to 7.44 kPa are included in Plate 10a, indicating that sand strength also has little influence on the torque versus load × radius relation at 20 percent slip.

72. Much less data scatter than in Plate 10a is present in Plate 10b, where the torque versus load × radius relation is shown for the 20 percent slip point of the programmed-increasing-slip tests in Table 8. Nearly all the data in Plate 10b lie inside the scatter band of Plate 10a. Overall, however, the data in Plate 10b show torque equal to only about 0.60 Wr, versus 0.67 Wr in Plate 10a. The smaller torque requirement in Plate 10b reflects the fact that values of p_b, s_w, RCG_h, dp_b, and t_{tb} for tests in Table 8 were generally more favorable to track pull performance than those for the tests in Plate 10a. (In particular, all tests in Table 8 had values of s_w, RCG_h, dp_b, and ds at the generally favorable values given in paragraph 38, and most had values of p_b and t_{tb} the same as listed there.)

73. Sinkage. The dependent variable chosen to characterize the during-test position of the track relative to the original sand surface at 20 percent slip is track sinkage beneath the rear road wheel (z_r). For practically all tests, the model track took a tail-down attitude at 20 percent slip, so that this sinkage closely approximated maximum track sinkage at that slip.

74. Plate 11 demonstrates that sinkage coefficient (z_R/ℓ) is closely associated with the sand-track mobility number, except for several tests with the 61.0- by 61.0-cm track (circled data points). More scatter of the sinkage data for this track than for the five others is considered to have resulted because the 1:1 width-to-length ratio causes the overall track to rotate about its center of gravity in a fashion dissimilar to that of tracks of 1:2, 1:4, and 1:8 ratios. Data from all tests in Tables 2-8 are included in Plate 11, so that the

relation shown there reflects the influence of a wide range of values for variables G, W, b, ℓ, p_b, s_w, RCG_h, dp_b, ds, and t_{tb}.

75. Trim angle. For the 20 percent slip condition, the model track assumed a tail-down attitude with deflection of the road bogies increasing front to rear. Thus, the angle of the track belt to the original soil surface (i.e. trim angle θ) took a different value between each set of deflected road bogies.* Rather than attempt to describe the trim angle pattern along the full length of a given track, it was judged more practical to use as a simple indicator of trim angle the term $\theta = \sin^{-1} z_R/\ell$. In effect, defining nominal trim angle this way assumes that the track belt is straight between the two end road bogies, and that the belt beneath the front bogie is just at the sand surface. Deviations from these idealized conditions were generally not large for the tests reported herein, and the relation of trim angle to the sand-track mobility number in Plate 11 can be used to predict track trim angle at 20 percent slip with reasonable accuracy.

Track performance at conditions other than 20 percent slip

76. The 20 percent slip point is important because near-maximum pull is obtained at this point in a situation in which actual track translational velocity (V_a) is still 80 percent of the theoretical track translational velocity ($V_t = r\omega$). Three other important performance levels from the pull-slip and torque-slip curves in Figure 6 are examined in the following paragraphs--the towed condition (net torque = 0), the self-propelled condition (pull = 0), and the maximum-tractive-efficiency condition (to be defined later). Results of the programmed-increasing-slip tests and the towed tests that are used in this examination are presented in Tables 8 and 9, respectively.

* A single value of track-belt trim angle was maintained for spans between adjacent pairs of road bogies only when those bogies developed zero deflection. Three or more consecutive zero bogie deflections generally were obtained only at the forward end of the longer track configurations (ℓ = 121.9 cm), where track-sand contact either did not occur or was so slight as to have very little influence on track performance.

77. Towed condition. The relations of the sand-track mobility number to towed force coefficient, slip, and sinkage coefficient (with nominal trim angle) are shown in Plates 12a, 12b, and 12c, respectively. Data from the zero net torque point of programmed-increasing-slip tests, as well as data from conventional towed tests, are used in each plate. The curves in Plates 12a, 12b, and 12c are each relatively flat for abscissa values larger than about 20. However, marked increases occur in the values of towed force coefficient (P_T/W), negative slip, sinkage coefficient of front road wheels, (z_F/ℓ), and negative trim angle as values of the mobility number decrease from 20 (as would be caused by increases in load and/or decreases in sand strength for a given track). Note that, for the towed condition, the front end of the track is lower than the rear (negative trim angle), so that maximum sinkage occurs under the front road wheel.

78. Self-propelled condition. The torque required for a track to develop zero pull, i.e. to just overcome sand motion resistance, is seen in Plate 13a to be closely related to the product of load times drive-sprocket radius (Wr). (All data in Plate 13 come from the programmed-increasing-slip tests in Table 8.) Thus, the amount of torque to enable a track to move forward under its own power in level sand appears to be relatively independent of track size, track shape, and soil strength (since a range of values of each of these variables is included in Plate 13).

79. Plates 13b and 13c show that, except for values of the sand-track mobility number smaller than about 20, values of slip, sinkage coefficient, and (nominal) trim angle for the self-propelled condition are quite small. Note that positive slip is required to overcome sand motion resistance and develop zero pull, and that a positive trim angle is developed for the self-propelled condition, with maximum sinkage at the rear road wheel.

80. Maximum-tractive-efficiency condition. A measurement of interest in describing practically any system that includes an input and an output is the efficiency with which the output is achieved. For the sand-track system, tractive efficiency (TE) is probably the

measure of greatest interest and is defined as

$$TE = \frac{\text{Output Power}}{\text{Input Power}}$$

which can be expressed as

$$TE = \frac{PV_a}{M\omega} = \left(\frac{PV_a}{M\frac{V_t}{r}}\right) \tag{5}$$

where

P = track pull

V_a = actual track translational velocity

M = torque input

ω = angular velocity of the drive sprocket

V_t = theoretical track translational velocity = $r\omega$

r = drive-sprocket pitch radius

By dividing both numerator and denominator by load W and substituting $V_a/V_t = 1 - S$ (where S = slip used as a decimal rather than as a percent), equation 5 becomes

$$TE = \frac{P/W}{M/Wr}(1 - S) \tag{6}$$

Equation 6 defines tractive efficiency in terms of the familiar pull and torque coefficients and a function of slip.

81. Another useful form for defining tractive efficiency is

$$TE = \frac{P}{W} \div \frac{M}{Wr\ (1 - S)} \tag{7}$$

where $\frac{M}{Wr\ (1 - S)}$ is termed the power number. A graphic illustration of how the value of tractive efficiency can be determined for a given programmed-increasing-slip test is shown in Figure 8, where pull coefficient (P/W) is plotted as a function of the power number. For positive pull, the slope of a line connecting the origin and a point on the curve defines tractive efficiency at that point. The shape of the curve in Figure 8 is typical--two long, nearly linear segments connected by a short curve. Note that beyond the maximum-tractive-efficiency (TE_{max}) point, very little gain in pull is obtained even for large increases in power number.

82. In describing the maximum-tractive-efficiency condition,

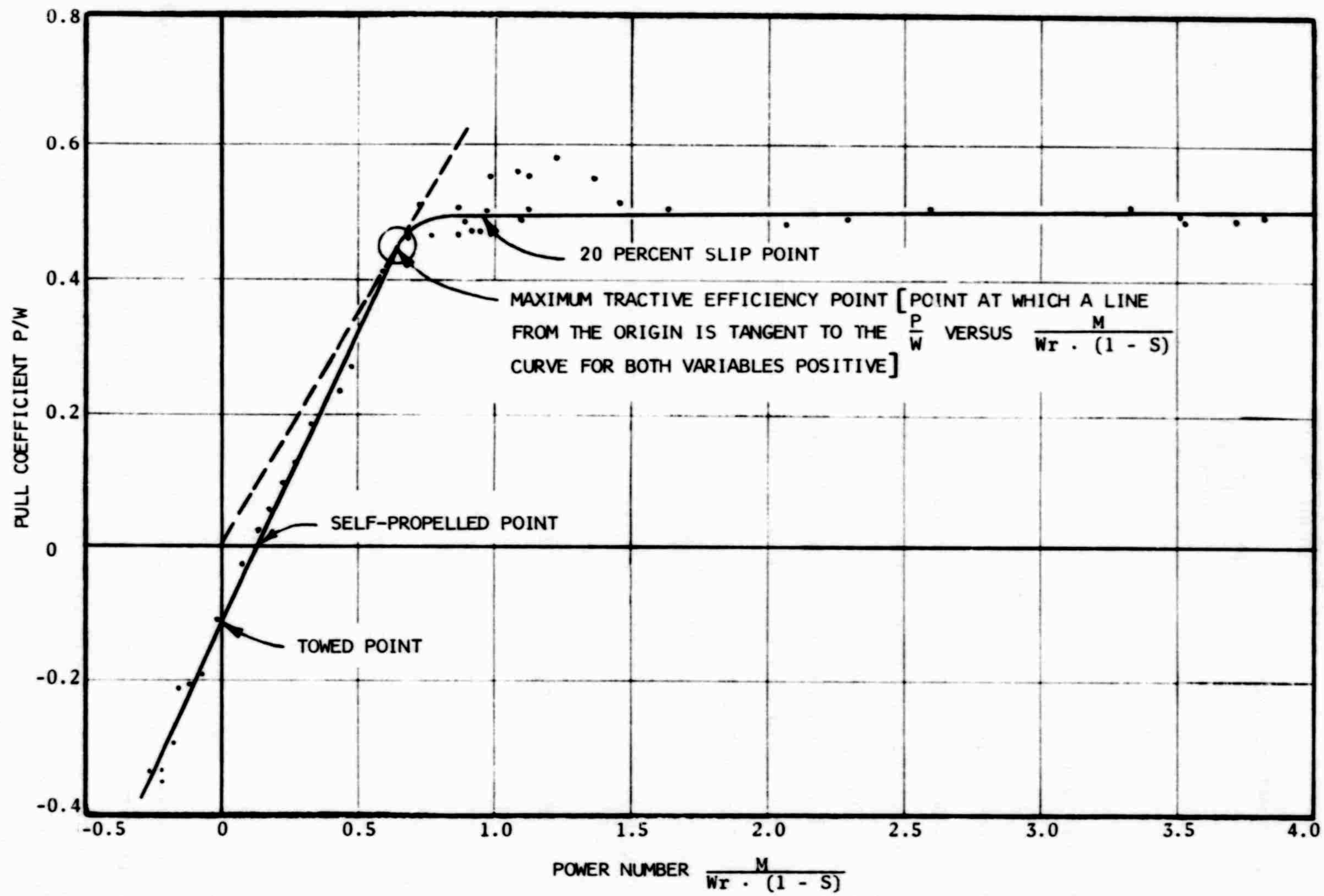

Figure 8. Obtaining trective efficiency from the pull coefficient versus power number curve for a programmed-increasing-slip test in Yuma sand

consider first three relations examined earlier for other performance conditions. As was the case at the 20 percent slip and the self-propelled points, net torque input is also closely related to load times drive-sprocket radius at the maximum-tractive-efficiency point (Plate 14a). As was true for the towed and self-propelled conditions, track slip is also closely associated with sand-track mobility number for the maximum-tractive-efficiency condition (Plate 14b). Finally, sinkage coefficient and (nominal) trim angle for that condition are closely related to the mobility number (Plate 14c).

83. Furthermore, tractive efficiency and pull coefficient for the maximum-tractive-efficiency condition are both closely related to the sand-track mobility number (Plates 15a and 15b, respectively). The similar curve shapes described by the two relations in Plate 15 come as no surprise when account is taken of equation 6 for tractive efficiency, together with the relations in Plates 14a and 14b. Plate 14a shows that torque coefficient (M/Wr) at the maximum-tractive-efficiency point is nearly a constant, and Plate 14b indicates that values of (1 - slip) at maximum-tractive-efficiency vary over only a small range for the full range of mobility number values considered. Thus, equation 6 is closely approximated by $\frac{P/W}{\text{constant}_1} \times \text{constant}_2$ for the maximum-tractive-efficiency condition, so that maximum-tractive-efficiency and pull coefficient at that maximum-efficiency should be related to the sand-track mobility number in similar fashion (Plate 15).

84. Of interest, too, is a comparison of the tractive efficiency and pull coefficient performance developed at maximum-tractive-efficiency versus that produced at 20 percent slip, a nominal slip value often used to characterize the near-maximum-pull condition. Plates 16a and16b show that, compared to maximum-tractive-efficiency, the 20 percent slip condition attains somewhat smaller values of tractive efficiency, but slightly larger values of pull coefficient for all except very small values of the sand-track mobility number--i.e., there is a trade-off in performance at maximum-tractive-efficiency and 20 percent slip points between tractive efficiency and pull coefficient. In both Plates 16a and 16b, differences between ordinate values for the maximum-

tractive-efficiency and the 20 percent slip curves are fairly small. Consider, also, that little increase in track pull is achieved at slip values larger than 20 percent (Figure 6), while the power required for this slight increase usually is excessive (Figure 8). Thus, 20 percent slip is a useful nominal value to describe near-maximum track pull in sand. A very desirable range of slip values for track operation in sand is that between maximum-tractive-efficiency and 20 percent slip, as indicated by the hatched area in Plate 16c.

Validation Tests

Tracked vehicle tests in sand

85. The ability of the standard relation of pull coefficient versus log of the sand-track mobility number (Plate 3)* to describe full-scale tracked-vehicle performance in air-dry sand was checked using results from tests of four vehicles--the M113A1 armored personnel carrier (APC), the M29C weasel, the M48A1 battle tank, and the CD4 engineer tractor. Pertinent characteristics of the test vehicles are included in Table 10.

86. Drawbar-pull tests were conducted with the above vehicles in air-dry mortar sand that was contained in a concrete-lined test pit 6.1-m wide, 1.9-m deep, and 54.9-m long. This pit is located adjacent to the one used for the model track tests reported herein. In each test, the test vehicle maintained a constant value of track speed or theoretical translational velocity (V_t). At the same time, a trailing vehicle, connected by a load cell and a cable to the test vehicle, steadily increased its braking force to cause the test vehicle values of actual forward velocity (v_a) to decrease uniformily from an initial

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* The term $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$ in Plate 3 is the same as the sand-track mobility number $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2} \cdot \left(\frac{d}{\ell/2}\right)^{n}$, since all data in Plate 3 had a value of $\frac{d}{\ell/2} = 1$.

positive value to zero. Thus, track slip was caused to vary from a low, self-propelled slip to 100 percent slip. Duplicate test runs were made for each vehicle-soil strength combination considered, and test results for the 20 percent slip condition are presented in Table 10.

Use of model-track relation to describe prototype-vehicle pull

87. For the M113A1, M29C, and M48A1, the value of the weight ratio (W/W_{max}) in the sand-track mobility number was taken as 0.4, since most existing tracked vehicles with flexible suspensions have been designed to this approximate ratio. Plate 17 shows that the standard curve from Plate 3 describes the pull coefficient performance of these vehicles quite well.

88. In Plate 17, data for the CD4 engineer tractor are described reasonably well by the standard relation with the weight ratio set equal to 1, as appropriate for a rigid, essentially girderized track. However, the trend of the CD4 data appears to be toward a lower value of pull coefficient than the standard curve for large values of the sand-track mobility number. The track assembly of the CD4 differs from those of the "cross-country" M113A1, M29C, and M48A1 vehicles and from the WES model track in that (a) its road wheels are mechanically restrained from deflecting; (b) its track is much more resistant to flexing; and (c) because its drive and idler sprockets are much larger, the diameter of each reaching from the bottom to the top of the track, the kinetics of individual track shoes while engaging and disengaging the soil are somewhat different. Further study is needed to evaluate how these factors influence track performance. Though based on a limited amount of data, the relation in Plate 17 is encouraging in that it indicates that the sand-track mobility number is a useful term for describing in-sand tracked vehicle performance, particularly for cross-country-type vehicles with flexible suspensions.

Summation

Major relations developed and some interpretations

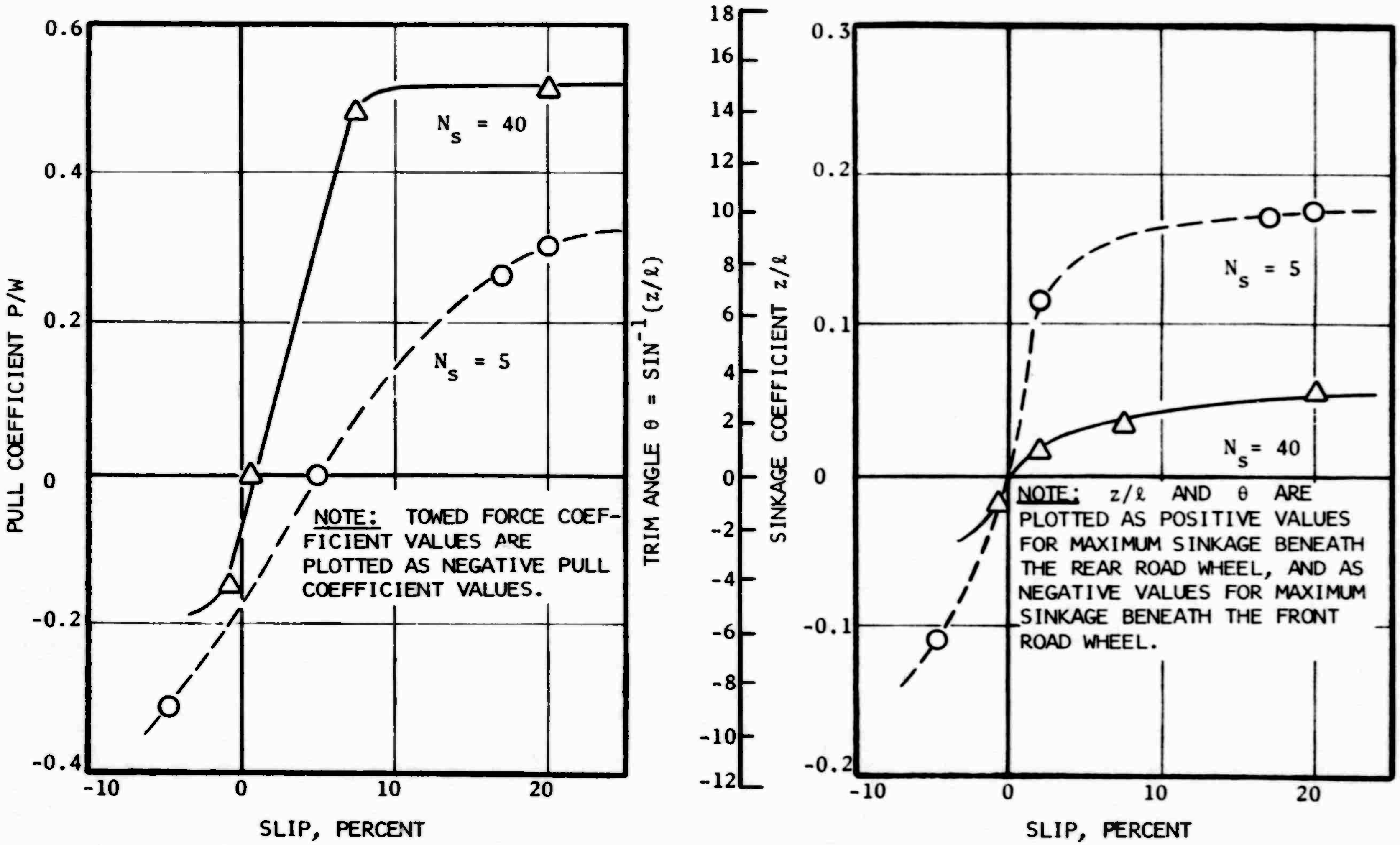
89. Results from laboratory tests of a versatile single model

track were analyzed to develop a comprehensive description of straight-line track performance at low speed in level test beds of air-dry sand. Performance was described in terms of variables pull, torque, slip, sinkage, and trim angle; quality of performance was described by tractive efficiency. Four performance levels were considered--the towed, self-propelled, 20-percent-slip, and maximum-tractive-efficiency conditions. Except for torque, each performance variable could be included in part of a dimensionless term and related to the dimensionless sand-track mobility number (equation 4). Torque was found to be insensitive to changes in the mobility number, but could be described from its relation to load times drive-sprocket radius. For the range of tests conditions considered, a detailed description of track performance was developed as illustrated in the following plates:

	Peformance Relations				
	Pull				
Performance Conditions	P/W vs N_s* or P/W vs log N_s	M vs Wr	Slip: Slip vs N_s	Tractive Efficiency: TE vs N_s	Sinkage and Trim Angle: z/ℓ & θ vs N_s
Towed	12a	--	12b	--	12c
Self-propelled	--	13a	13b	--	13c
Maximum-Tractive-Efficiency	15b	14a	14b	15a	14c
20 Percent Slip	3,16b	10a	--	16a	11

90. A considerable amount of information can be extracted from the above plates. For example, if the track and sand strength characteristics of a particular situation are known (i.e., if the value of the sand-track mobility number N_s is known), then the curves in plates listed under "pull", "slip," and "sinkage and trim angle" can be used to estimate the relations of slip to pull coefficient, sinkage coefficient, and trim angle for the particular mobility number value of interest. Examples of such estimates are presented in Figure 9 for two mobility

* Equation 4.



a. Pull coefficient versus slip

b. Sinkage coefficient and trim angle versus slip

Figure 9. Sketches of slip versus pull coefficient, sinkage coefficient, and trim angle for two values of sand-track mobility number N_S

number values, 5 and 40. Note in Figure 9a that the shape of the pull-slip curve becomes more angular as the value of the mobility number increases; i.e., for increasing values of the mobility number, the transition from the towed to the maximum-tractive-efficiency condition occurs within an ever-narrowing range of slip values, and the slope of the pull-slip curve outside the towed and maximum-tractive-efficiency points becomes progressively smaller. Ordinate values along the full length of the pull-slip curve increase as values of the mobility number increase. In Figure 9b, the S-shaped trim angle versus slip curves passing through the origin are typical of those obtained for tracks with the at-rest center of gravity at the geometric center line and with the trim angle unrestrained.[12] Ordinate values of such curves can be expected to increase (in both the positive and negative directions) as values of the mobility number decrease (i.e. as values of penetration resistance gradient decrease and/or values of load increase for a given track size).

91. From the plates listed in paragraph 89 under "torque", the amount of net torque required to progress from the towed to the self-propelled condition is about 0.110 Wr,* to the maximum-tractive-efficiency condition about 0.540 Wr, and to the 20 percent slip condition about 0.670 Wr. Also, it is possible to estimate the tractive efficiency versus slip relation for a given value of the sand-track mobility number using equation 7 and values extracted from the curves of torque versus load times drive-sprocket ratio and pull coefficient and slip versus the mobility number in the plates in paragraph 89.

Applying the major relations to real world problems

92. The relations cited above, as well as many others developed in this report, are useful in describing quantitatively the behavior of tracks in air-dry sand. Generally speaking, however, tracked vehicles operating alone, straight-line, on level sand simply do not develop

* The straight-line of slope 0.110 used in Plate 13a to describe the torque versus Wr relation for the self-propelled condition agrees with the straight-line interpretations made for the maximum-tractive-efficiency and 20 percent slip conditions (Plates 14a and 10a, respectively). Examination of Plate 13a suggests, however, that a curved line might provide a slightly better fit to the data.

severe mobility problems. Consider, for example, that the smallest value of pull coefficient developed at 20 percent slip for any of the tests in this report is 0.105, even though tests included track contact pressures ($W/b\ell$ values) up to 200 kPa and sand penetration resistance values as small as 0.50 Mn/m^3. This is not to say, however, that tracked vehicles do not encounter very real mobility problems in air-dry sand. Consider, for example, that the tangent of the maximum slope climbable by a tracked vehicle in dry, loose sands can be taken as numerically equal to maximum pull coefficient on level ground minus 10 percent[13] (where maximum pull coefficient is closely estimated by pull coefficient at 20 percent slip). Thus, even moderate slopes can immobilize tracked vehicles in loose, air-dry sand.

93. Also, situations arise where a tracked vehicle must tow another vehicle in sand. A no-go situation occurs when maximum pull available (closely approximated by pull at 20 percent slip--see Plate 3 or 16b) is equal to or less then the towing force required (estimated as towed force from Plate 12a for tracked vehicles, or from Plate 22 of Reference 14 for wheeled vehicles). Examination of these plates reveals that no-go can occur for this type operation (but on level sand usually only at very small values of the sand-track mobility number).

94. Of course, all relations herein of the form (dimensionless performance term) versus (sand-track mobility number N_s) can be used either to (a) estimate performance from a known or estimated value of the mobility number, or (b) choose or design a loaded track such that, with a known or estimated value of sand strength, a value of the mobility number is produced that corresponds to the performance level required. This flexibility, together with the fact that all of the relations and variables cited in paragraph 89 are easily described in quantitative form, makes the relations in that paragraph powerful tools in describing track performance in air-dry sand.

PART V: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

95. The foregoing analysis is considered adequate basis for the following conclusions:

a. The 26 independent soil and single-track variables and 5 dependent (performance) variables considered herein provide a reasonably comprehensive framework within which in-soil track performance can be described (paragraphs 19 and 20).

b. From the full listings of 23 and 5 dimensionless Pi terms that describe the independent and dependent variables, respectively, of the soil-single track system (paragraphs 21-25), only 15 and 5 Pi terms, respectively, were needed to describe slow-speed, straight-line performance in flat, level, air-dry sand (paragraphs 26-31).

c. Plackett-Burman (statistical) screening tests determined that 7 independent variables included in the 15 independent Pi terms mentioned under b above have the most effect on pull at 20 percent slip: load (W), sand penetration resistance gradient (G), track width (b), track-hard surface contact length (ℓ), hard-surface road-bogie deflection (Δ), spacing between road wheels (s_w), and horizontal at-rest center of gravity (RCG_h), (paragraphs 32-35).

d. The basic-variable prediction term (equation 1) developed in Report 2[2] was expanded to the sand-track mobility number N_s (equation 4), which takes into account the influence on the track pull coefficient performance of all seven independent variables under c above (paragraphs 38-51).

e. Among those independent variables included in the 15 independent Pi terms of b above, but not mentioned under c or d, only three were determined to influence track in-sand pull coefficient significantly: distribution of pressure in road-bogie cylinders (dp_b), drive-sprocket location (ds), and index of track-belt tension (t_{tb}) (paragraphs 52-58).

f. The sand-track mobility number can be adjusted to take into account that better track pull coefficient performance results if road-bogie cylinder pressure decreases, rather than increases, from front-to-rear of track (i.e. if resistance to bogie deflection decreases from front to rear) (paragraphs 63-65). Slightly better performance results if tension in the track belt is maintained at a high level (paragraphs 60-62) and if the drive sprocket is located at the rear of the track (paragraphs 66-67).

g. The sand-track mobility number is closely related to dimensionless performance term pull/load at the towed, maximum-tractive-efficiency (TE_{max}), and 20 percent slip points (paragraphs 77, 83, and 43, respectively); to track slip at the towed, self-propelled (zero pull) and TE_{max} points (paragraphs 77, 79, and 82, respectively); and to tractive efficiency (equation 7) at the TE_{max} and 20 percent slip points (paragraphs 83 and 84).

h. Torque is not closely related to the sand-track mobility number, but can be described from its relation to the product load times drive-sprocket radius at the self-propelled, TE_{max}, and 20 percent slip points (paragraphs 78, 82, and 69-72, respectively).

i. Sinkage coefficient (z/ℓ) and nominal trim angle ($\theta = \sin^{-1} z/\ell$) are closely associated with the mobility number at each of the towed, self-propelled, TE_{max}, and 20 percent slip points (paragraphs 77, 79, 82, and 73-75, respectively). With track RCG_h at center line, maximum sinkage occurs beneath the front road wheel (trim angle negative) for the towed condition, and beneath the rear road wheel (trim angle positive) for the self-propelled, TE_{max}, and 20 percent slip conditions.

j. The 20 percent slip condition develops slightly more pull at slightly less tractive efficiency than does the TE_{max} condition. Increasing slip beyond about 20 percent increases pull very slightly, with an accompanying large decrease in tractive efficiency. Thus, 20 percent slip is a useful nominal value to characterize near-maximum track pull in air-dry sand (paragraph 84).

k. Results from in-sand laboratory tests of four full-size tracked vehicles indicate that the model-developed sand-track mobility number can be used to describe prototype vehicle performance (paragraphs 85-88).

l. A comprehensive set of relations was developed that allows performance (in terms of pull, torque, slip, sinkage, trim angle, and tractive efficiency) to be predicted at the towed, self-propelled, TE_{max}, and 20 percent slip conditions for a wide range of sand-track conditions. Reversing the order, these relations allow a track to be chosen or designed to satisfy a particular in-sand performance requirement. Though developed from level-ground tests of a single track, the pull/load versus mobility number relations can easily be extrapolated to slope-climbing or vehicle-towing situations (paragraphs 89-94).

Recommendations

96. It is recommended that:

a. A much smaller track be fabricated to study the effects on performance of nonuniform soil strength profiles and various track-shoe shapes and spacings.

b. Field tests be conducted in sand with several prototype tracked vehicles to validate the laboratory-developed sand-track mobility number, and to modify it, if necessary, to account for the effects of an extended range of conditions with respect to vehicle speed, vehicle maneuvering (steering), sandy soil type, soil strength profiles, ground slope, track shoe shapes and spacings, and vehicle configurations (including articulated vehicles).

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Table 1

Ranges of Possible Test Values for the Independent Sand and Single-Track Variables

Variable	Symbol	Mass, Length, Time (MLT) Units	Range of Values
Load	W	MLT^{-2}	Near zero to 27 kN
Soil strength	G	$ML^{-2}T^{-2}$	0.5 to 7.5 MN/m^3 (approximately)
Track width	b	L	15.2, 30.5, and 61.0 cm
Track contact length (on a hard surface)	ℓ	L	61.0 and 121.9 cm
Angle of approach	α	-	5.5 to 18.5 deg with forward end bogie fully retracted; 21.5 to 33.0 deg with it fully extended
Angle of departure	β	-	Same ranges as for α
Horizontal location of at-rest center of gravity	RCG_h	L/L	Up to 0.3ℓ forward and rearward of center line
Vertical location of at-rest center of gravity	RCG_v	L/L	Center of load axle to approximately 4 cm above center
Road-wheel diameter	d_w	L	17.8 cm only with present system
Road-wheel spacing (uniform with at least one inner road wheel)	s_w	L	For ℓ = 61.0 cm: 20.3 cm only. For ℓ = 121.9 cm: 20.3, 40.6, and 61.0 cm
(Pneumatic) pressure in each road bogie	p_b	$ML^{-1}T^{-2}$	0 to 621 kPa
Distribution of pressure in road bogies	dp_b	$f(ML^{-1}T^{-2})$	Wide variety possible
Drive-sprocket location	ds	-(or L/L)	Front or rear (or can be described in terms of geometric location relative to some fixed point on the track)
Track-shoe height	h_s	L	1.3, 2.5, and 5.1 cm
Track-shoe thickness	th_s	L	0.32 and 0.64 cm
Track-shoe spacing	s_s	L	3.0 cm (all shoes in); 14.2 cm (every other shoe removed)
Index of track-belt tension	t_{tb}	$ML^{-1}T^{-2}$	Gage range 0 to 20,700 kPa; values of 1380 to 6890 kPa have been tested
Actual track unit translational velocity	V_a	LT^{-1}	0 to 0.6 m/sec
Slip*	S	-	-100 to +100 percent
Track trim angle	θ	-	0 to 20 deg
Drive sprocket pitch radius	r	L	16.51 cm only with present system
Other pertinent track dimensions (such as maximum bogie deflection, additional track shoe measurements, etc.)	ℓ'	L	Several other possibly important linear dimensions can be closely controlled and measured

* Track slip, in percent, is defined as $S = [(V_t - V_a)/V_t] \times 100$, where V_t is theoretical track translational velocity (i.e. $r\omega$, where r is drive sprocket pitch radius and ω is angular velocity of the drive sprocket) and V_a is actual track unit translational velocity (i.e. the translational velocity of the overall dynamometer carriage). In practice, slip is controlled and measured via control over the values of V_t and V_a. The influence of variable V_t on the soil-track system is rarely considered independently, but is considered within the context that V_t (together with V_a) determine slip, which definitely affects track performance.

Table 2

Tests of Single Tracks in Mortar Sand, 20 Percent Slip, Fir[illegible]
Road-Bogie Cylinder Pressure Uniform over Track
Length at 276-, 448-, and 621-kPa

Test No.	Track Width b, cm	Track Length ℓ cm	Pen. Resist. Gradient G, MN/m^3	Load W, N	Slip S, %	Index of Track Belt Tension t_{tb}, kPa	Pressure (kPa) in Bogie No.* 1	2	3	4	5	6	7	Deflection (δ) in cm of Bogie No. 1	2	3	4	5	6	7	Net Torque M, m-N	Pull P, N	Sinkage At Rear Bogie z_R, cm
Pressure in Road Bogies Initially Set at 276 kPa																							
D-71-0073-2	15.2	61.0	1.04	8,385	18.9	6087	324	324	293	278	-	-	-	7.0	3.2	1.7	1.0	-	-	-	1230	2245	12.3
74			1.07	5,017	20.8	6687	323	284	276	280	-	-	-	4.4	1.8	0.0	0.2	-	-	-	626	1574	13.5
75			1.11	2,775	20.3	5714	314	276	277	272	-	-	-	1.9	0.0	0.0	0.0	-	-	-	430	1070	13.3
91	30.5	61.0	1.14	2,608	22.0	6457	302	277	276	-	-	-	-	2.8	0.0	0.0	0.0	-	-	-	213	1230	6.6
92			1.20	4,862	16.6	5826	294	290	279	-	-	-	-	5.1	1.1	0.0	0.0	-	-	-	502	2340	7.2
93			1.23	8,479	22.9	4774	294	312	278	-	-	-	-	7.5	3.2	0.7	0.0	-	-	-	789	3492	4.5
35	15.2	121.9	1.02	8,198	19.7	6936	294	-	283	-	270	-	275	4.7	2.3	0.8	0.0	0.0	0.0	0.0	1131	2760	16.1
36			1.05	13,499	18.8	6927	321	-	316	-	275	-	276	8.8	5.4	2.9	0.9	0.0	0.0	0.0	1531	4890	17.5
46	30.5	121.9	1.03	5,760	24.7	5959	302	-	288	-	275	-	275	3.2	1.9	0.5	0.0	0.0	0.0	0.0	594	2715	9.3
47			1.02	4,801	23.2	6213	314	-	281	-	278	-	272	2.5	0.9	0.3	0.0	0.0	0.0	0.0	521	2348	9.0
48			1.15	15,590	18.8	4715	315	-	318	-	271	-	268	8.9	8.4	6.1	3.3	1.2	0.0	0.0	1481	7622	5.3
49			1.16	12,012	20.4	5240	331	-	318	-	272	-	270	8.6	6.9	4.4	1.7	0.0	0.0	0.0	1402	6063	7.3
50			0.98	8,858	21.3	5500	337	-	319	-	273	-	272	5.9	5.9	2.8	0.6	0.0	0.0	0.0	1039	4506	11.2
55	61.0	121.9	1.18	14,628	22.6	3952	317	-	324	-	292	-	274	10.5	8.2	5.8	2.9	0.9	0.0	0.0	1703	8028	4.1
56			1.19	17,039	20.5	-	322	-	316	-	274	-	274	6.5	4.8	2.7	0.4	0.0	0.0	0.0	2183	9165	6.6
Pressure in Road Bogies Initially Set at 448 kPa																							
D-71-0070-2	15.2	61.0	1.09	2,697	16.5	4940	484	448	449	448	-	-	-	1.2	0.0	0.0	0.0	-	-	-	374	811	12.8
71			1.11	4,353	16.8	6890	488	447	448	447	-	-	-	2.7	0.0	0.0	0.0	-	-	-	571	1349	14.2
72			1.10	6,636	19.8	4951	494	449	450	449	-	-	-	3.8	0.0	0.0	0.0	-	-	-	960	1670	15.9
76			1.19	14,767	16.3	5972	481	488	451	447	-	-	-	7.8	3.0	1.2	0.5	-	-	-	1925	2492	14.9
77			1.14	11,359	17.4	6710	495	482	451	450	-	-	-	6.1	1.3	0.0	0.0	-	-	-	1533	2058	17.6
78			1.16	8,195	17.3	6605	494	454	447	448	-	-	-	4.9	0.0	0.0	0.0	-	-	-	1180	2046	16.5
88	30.5	61.0	1.16	14,956	22.2	4852	484	469	460	-	-	-	-	10.4	8.7	9.1	8.1	-	-	-	1425	5096	6.2
89			1.12	8,324	20.1	5219	474	478	454	-	-	-	-	5.5	0.8	0.0	0.0	-	-	-	720	3246	10.1
90			1.14	3,207	20.0	5692	468	447	447	-	-	-	-	2.6	0.0	0.0	0.0	-	-	-	338	1353	8.6
31	15.2	121.9	0.97	17,617	19.2	6879	512	-	476	-	449	-	461	6.9	3.6	1.4	0.0	0.0	0.0	0.0	2080	5195	22.2
32			1.00	10,284	20.9	6894	473	-	449	-	451	-	450	1.4	0.0	0.0	0.0	0.0	0.0	0.0	1140	2731	20.1
41	30.5	121.9	1.06	5,934	23.1	4735	465	-	447	-	446	-	446	1.1	0.0	0.0	0.0	0.0	0.0	0.0	641	2796	9.8
42			1.04	9,899	24.0	4684	473	-	453	-	449	-	448	2.3	0.5	0.3	0.0	0.0	0.0	0.0	953	3799	11.6
43			1.18	14,250	22.2	5126	485	-	488	-	446	-	447	6.0	3.7	2.4	0.1	0.0	0.0	0.0	1274	5324	11.5
44			1.17	18,306	19.2	4533	490	-	489	-	447	-	447	9.0	5.9	3.4	1.1	0.0	0.0	0.0	1745	7687	10.3
45			1.17	22,943	16.5	4448	494	-	493	-	455	-	449	10.6	7.3	4.7	2.1	0.4	0.5	0.4	2199	9295	9.8
60	61.0	121.9	1.04	22,011	21.7	3047	484	-	492	-	462	-	453	10.6	7.9	5.5	2.6	0.6	0.0	0.5	2206	10413	5.5
Pressure in Road Bogies Initially Set at 621 [illegible]																							
D-71-0067-2	15.2	61.0	1.15	8,001	17.2	6524	639	620	620	620	-	-	-	3.2	0.0	0.0	0.0	-	-	-	1112	1880	17.[illegible]
68			1.07	5,511	16.2	4668	664	621	622	620	-	-	-	1.9	0.0	0.0	0.0	-	-	-	752	1488	16.[illegible]
69			1.09	2,717	10.4	6077	630	620	621	620	-	-	-	0.1	0.0	0.0	0.0	-	-	-	329	726	13.[illegible]
79			0.98	11,693	17.5	6661	631	618	618	619	-	-	-	4.1	0.0	0.0	0.0	-	-	-	1708	2252	21.[illegible]
80			1.02	14,302	19.5	6813	633	627	616	615	-	-	-	4.8	0.4	0.0	0.0	-	-	-	2050	2374	21.[illegible]
81			1.04	17,135	17.6	6469	636	645	617	616	-	-	-	6.0	1.2	0.0	0.0	-	-	-	2264	2639	21.[illegible]
86	30.5	61.0	1.02	12,536	19.0	4887	650	656	623	620	-	-	-	5.4	1.0	0.0	0.0	-	-	-	1121	3645	11.[illegible]
87			1.03	17,529	15.8	4207	647	657	620	620	-	-	-	7.1	2.5	0.0	0.0	-	-	-	1490	4709	9.[illegible]
98	61.0	61.0	1.23	12,360	16.3	5947	660	661	621	-	-	-	-	5.9	1.2	0.1	0.0	-	-	-	1100	5069	8.[illegible]
99			1.30	17,342	19.1	4777	653	672	624	-	-	-	-	7.4	2.6	0.0	0.0	-	-	-	1574	6518	6.[illegible]
15	15.2	121.9	3.92	3,131	23.1	6876	622	-	620	-	620	-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	401	1706	6.[illegible]
27			5.71	5,245	23.5	6880	621	-	620	-	596	-	619	0.0	0.0	0.0	0.0	0.0	0.0	0.0	496	2593	6.[illegible]
29			1.05	10,102	21.0	6906	614	-	619	-	624	-	618	0.1	0.0	0.0	0.0	0.0	0.0	0.0	1161	2822	21.[illegible]
30			1.11	18,790	18.3	6899	670	-	621	-	622	-	619	3.9	1.4	0.2	0.0	0.0	0.0	0.0	2009	4088	16.[illegible]
37	30.5	121.9	0.97	25,594	18.4	2477	652	-	660	-	620	-	619	8.8	5.8	3.7	1.5	0.0	0.0	0.0	2253	8768	11.[illegible]
38			0.99	15,721	21.3	4678	646	-	629	-	631	-	625	1.8	0.4	0.3	0.0	0.0	0.0	0.0	1334	4906	14.[illegible]
39			0.97	5,342	25.4	6467	622	-	617	-	559	-	616	0.2	0.0	0.0	0.0	0.0	0.0	0.0	569	2717	8.[illegible]
61	61.0	121.9	0.96	25,593	15.8	3235	646	-	659	-	614	-	617	9.5	6.8	4.5	1.8	0.0	0.0	0.0	2133	11437	[illegible]
62			0.99	15,002	23.4	5124	644	-	637	-	641	-	619	2.1	0.8	0.7	0.0	0.0	0.0	0.0	1695	6475	7.[illegible]
63			0.97	4,896	24.0	6469	621	-	620	-	626	-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	736	2877	[illegible]

* Road bogies are numbered consecutively, starting at the rear end of the track.

** Here trim angle θ is defined as $\theta = \sin^{-1}(z_R/\ell)$.

† $\left(\frac{d}{\ell/2}\right)^n$ is not included as part of the sand-track mobility number (equation 4) for tests in this table because for each test $\frac{d}{\ell/2}$ had a value of 1.0.

Table 2

Tracks in Mortar Sand, 20 Percent Slip, First Pass
Bogie Cylinder Pressure Uniform over Track
Length at 276-, 448-, and 621-kPa

cm of Bogie No. 4	5	6	7	Net Torque M, m-N	Pull P, N	Sinkage At Rear Bogie z_R, cm	Trim** Angle θ, deg	Basic-Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$	Sand-Track Mobility Number† $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$	Pull Coefficient $\frac{P}{W}$	Torque Coefficient $\frac{M}{Wr}$	Torque Predictor Wr, m-N	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1-S)$	Sinkage Coefficient $\frac{z_R}{\ell}$
e in Road Bogies Initially Set at 276 kPa														
.0	-	-	-	1230	2245	12.3	11.7	3.5	3.4	0.268	0.888	1384	0.245	0.202
.2	-	-	-	626	1574	13.5	12.8	6.0	4.5	0.314	0.756	828	0.329	0.221
.0	-	-	-	430	1070	13.3	12.6	11.3	6.3	0.386	0.939	458	0.328	0.218
.0	-	-	-	213	1230	6.6	6.2	35.1	19.0	0.472	0.495	431	0.744	0.108
.0	-	-	-	502	2340	7.2	6.8	19.8	14.6	0.481	0.625	803	0.642	0.118
.0	-	-	-	789	3492	4.5	4.2	11.6	11.3	0.412	0.564	1400	0.563	0.074
.0	0.0	0.0	0.0	1131	2760	16.1	8.5	9.9	7.2	0.337	0.836	1353	0.324	0.148
.9	0.0	0.0	0.0	1531	4890	17.5	8.3	6.2	5.8	0.362	0.687	2229	0.428	0.144
.0	0.0	0.0	0.0	594	2715	9.3	4.4	40.5	24.6	0.471	0.625	951	0.567	0.076
.0	0.0	0.0	0.0	521	2348	9.0	4.2	48.2	26.7	0.489	0.657	793	0.572	0.074
.3	1.2	0.0	0.0	1481	7622	5.3	2.5	16.7	16.7	0.489	0.575	2574	0.691	0.043
.7	0.0	0.0	0.0	1402	6063	7.3	3.4	21.9	19.2	0.505	0.707	1983	0.569	0.060
.6	0.0	0.0	0.0	1039	4506	11.2	5.3	25.1	18.9	0.375	0.710	1462	0.416	0.092
.9	0.9	0.0	0.0	1703	8028	4.1	1.9	51.7	50.0	0.549	0.705	2415	0.603	0.034
.4	0.0	0.0	0.0	2183	9165	6.6	3.1	44.8	44.8	0.538	0.776	2813	0.551	0.054
e in Road Bogies Initially Set at 448 kPa														
.0	-	-	-	374	811	12.8	12.1	11.2	4.8	0.301	0.840	445	0.299	0.210
.0	-	-	-	571	1349	14.2	13.5	7.1	3.9	0.310	0.795	719	0.324	0.233
.0	-	-	-	960	1670	15.9	15.1	4.6	3.1	0.252	0.876	1096	0.258	0.261
.5	-	-	-	1925	2492	14.9	14.1	2.2	2.2	0.169	0.790	2438	0.179	0.244
.0	-	-	-	1533	2058	17.6	16.8	2.8	2.5	0.181	0.817	1875	0.183	0.289
.0	-	-	-	1180	2046	16.5	15.7	3.9	2.9	0.250	0.872	1353	0.237	0.270
.1	-	-	-	1425	5096	6.2	5.9	6.2	6.2	0.341	0.577	2469	0.460	0.102
.0	-	-	-	720	3246	10.1	9.6	10.8	8.2	0.390	0.524	1374	0.595	0.166
.0	-	-	-	338	1353	8.6	8.1	28.5	13.4	0.422	0.638	529	0.529	0.141
.0	0.0	0.0	0.0	2080	5195	22.2	10.5	4.3	3.6	0.295	0.715	2909	0.333	0.182
.0	0.0	0.0	0.0	1140	2731	20.1	9.5	7.6	4.8	0.266	0.671	1698	0.313	0.165
.0	0.0	0.0	0.0	641	2796	9.8	4.6	40.5	19.6	0.471	0.654	980	0.554	0.080
.0	0.0	0.0	0.0	953	3799	11.6	5.4	23.8	14.9	0.384	0.583	1634	0.501	0.095
.1	0.0	0.0	0.0	1274	5324	11.5	5.4	18.8	14.1	0.374	0.542	2353	0.537	0.094
.1	0.0	0.0	0.0	1745	7687	10.3	4.8	14.5	12.3	0.420	0.577	3022	0.588	0.084
.1	0.4	0.5	0.4	2199	9295	9.8	4.6	11.6	11.0	0.405	0.581	3788	0.582	0.080
.6	0.6	0.0	0.5	2206	10413	5.5	2.6	30.3	28.2	0.473	0.607	3634	0.610	0.045
ssure in Road Bogies Initially Set at 621 kPa														
.0	-	-	-	1112	1880	17.5	16.7	4.0	2.5	0.235	0.842	1321	0.231	0.287
.0	-	-	-	752	1488	16.2	15.4	5.4	2.8	0.270	0.826	910	0.274	0.266
.0	-	-	-	329	726	13.6	12.9	11.1	4.1	0.267	0.733	449	0.326	0.223
.0	-	-	-	1708	2252	21.1	20.2	2.3	1.8	0.193	0.885	1931	0.180	0.346
.0	-	-	-	2050	2374	21.9	21.0	2.0	1.7	0.166	0.868	2361	0.154	0.359
.0	-	-	-	2264	2639	21.8	20.9	1.7	1.6	0.154	0.800	2829	0.159	0.357
.0	-	-	-	1121	3645	11.0	10.4	6.5	5.1	0.291	0.542	2070	0.435	0.180
.0	-	-	-	1490	4709	9.8	9.3	4.7	4.4	0.269	0.515	2894	0.440	0.161
.0	-	-	-	1100	5069	8.1	7.6	22.6	17.7	0.410	0.539	2041	0.637	0.133
.0	-	-	-	1574	6518	6.1	5.7	17.0	15.8	0.376	0.550	2863	0.554	0.100
.0	0.0	0.0	0.0	401	1706	6.0	2.8	97.9	29.2	0.545	0.776	517	0.540	0.049
.0	0.0	0.0	0.0	496	2593	6.3	3.0	85.1	32.8	0.494	0.573	866	0.660	0.052
.0	0.0	0.0	0.0	1161	2822	21.0	9.9	8.1	4.3	0.280	0.696	1668	0.318	0.172
.0	0.0	0.0	0.0	2009	4088	16.0	7.5	4.6	3.4	0.218	0.648	3102	0.275	0.131
.5	0.0	0.0	0.0	2253	8768	11.5	5.4	8.6	7.3	0.343	0.533	4226	0.525	0.094
.0	0.0	0.0	0.0	1334	4906	14.9	7.0	14.3	9.6	0.312	0.514	2596	0.478	0.122
.0	0.0	0.0	0.0	569	2717	8.3	3.9	41.2	16.1	0.509	0.645	882	0.589	0.068
.8	0.0	0.0	0.0	2133	11437	6.6	3.1	24.1	20.5	0.447	0.505	4225	0.745	0.054
.0	0.0	0.0	0.0	1695	6475	7.6	3.6	42.2	27.5	0.431	0.684	2482	0.483	0.062
.0	0.0	0.0	0.0	736	2877	6.0	2.8	127.0	47.3	0.588	0.911	808	0.491	0.049

for each test $\frac{d}{\ell/2}$ had a value of 1.0.

Test No.	Track Width b cm	Track Length ℓ cm	Pen. Resist. Gradient $G, MN/m^3$	Load W, N	Slip S, %	Index of Track Belt Tension t_{tb}, kPa	Pressure (kPa) in Bogie No.*							Def
							1	2	3	4	5	6	7	1
D-71-0067-1	15.2	121.9	1.06	4,420	23.7	6903	620	-	620	-	620	-	621	0.0
68			1.09	10,488	20.0	6883	622	-	620	-	620	-	622	4.2
69			1.16	18,317	22.0	6898	612	-	619	-	619	-	623	10.1
76	30.5	121.9	1.01	4,466	22.2	6712	617	-	620	-	619	-	620	0.1
77			0.99	11,091	23.9	4390	619	-	619	-	620	-	620	6.7
78			1.04	18,042	20.0	3330	616	-	618	-	612	-	620	10.4
79	61.0	121.9	1.13	18,367	22.9	3542	616	-	618	-	615	-	620	10.3
80			1.11	10,731	24.1	4032	617	-	618	-	619	-	620	6.6
81			1.09	5,053	23.0	6219	619	-	620	-	619		620	0.8
D-71-0070-1	15.2	121.9	1.33	7,786	23.6	5450	618	-	-	NM††	-	-	648	6.2
71			1.04	4,510	24.6	6082	618	-	-	NM	-	-	620	2.5
72			1.27	1,841	24.2	6920	617	-	-	NM	-	-	620	0.1
73	30.5	121.9	1.01	2,409	24.8	6710	621	-	-	NM	-	-	621	0.0
74			1.07	4,785	24.5	5768	618	-	-	NM	-	-	622	2.0
75			1.11	7,538	24.8	5152	618	-	-	NM	-	-	608	4.9
82	61.0	121.9	0.96	2,693	23.5	6739	619	-	-	NM	-	-	620	0.0
83			0.97	5,097	24.6	5540	616	-	-	NM	-	-	620	2.5
84			1.03	7,496	21.9	4789	619	-	-	NM	-	-	620	4.6

* Road bogies are numbered consecutively, starting at the <u>rear</u> end of the track.

** Here, trim angle θ is defined as $\theta = \sin^{-1}(z_R/\ell)$.

† $\left(\frac{d}{\ell/2}\right)^n$ is not included as part of the sand-track mobility number (equation 4) for tests in t

†† NM means "not measured."

Table 3

Tests of Single Tracks in Yuma Sand, 20 Percent Slip, First Pass
40.6- and 61.0-cm Road-Wheel Spacings

) in Bogie No.* 4	5	6	7	Deflection (δ) in cm of Bogie No. 1	2	3	4	5	6	7	Net Torque M,m-N	Pull P, N	Sinkage at Rear Bogie z_R,cm	Trim** Angle θ,deg	Basic-Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$	San Mob Num $\frac{G(b\ell)^{3/2}}{W}$
											Road Wheel Spacing = 40.6 cm					
-	620	-	621	0.0	-	0.0	-	0.0	-	0.0	477	1608	15.0	7.1	18.8	
-	620	-	622	4.2	-	0.6	-	0.0	-	0.0	1352	2804	20.9	9.8	8.1	
-	619	-	623	10.1	-	5.0	-	0.3	-	0.0	2461	5016	18.7	8.9	5.0	
-	619	-	620	0.1	-	0.0	-	0.0	-	0.0	408	2163	9.8	4.6	51.3	
-	620	-	620	6.7	-	3.3	-	0.0	-	0.0	1045	4094	12.9	6.1	20.2	
-	612	-	620	10.4	-	6.8	-	8.7	-	0.0	1944	7202	6.3	3.0	12.8	
-	615	-	620	10.3	-	7.0	-	2.8	-	0.0	1817	7504	4.9	2.3	39.5	
-	619	-	620	6.6	-	3.2	-	0.0	-	0.0	1066	4298	9.5	4.5	66.3	
-	619		620	0.8	-	0.0	-	0.0	-	0.0	460	2565	9.7	4.6	138.3	
											Road Wheel Spacing = 61.0 cm					
NM††	-	-	648	6.2	-	-	0.0	-	-	0.0	1053	2696	18.7	8.8	13.4	
NM	-	-	620	2.5	-	-	0.1	-	-	0.0	625	1507	16.1	7.6	18.0	
NM	-	-	620	0.1	-	-	0.0	-	-	0.0	167	1012	11.9	5.3	53.9	
NM	-	-	621	0.0	-	-	0.0	-	-	0.0	225	1206	8.4	4.0	95.0	
NM	-	-	622	2.0	-	-	0.0	-	-	0.0	496	2157	10.4	4.9	50.7	
NM	-	-	608	4.9	-	-	0.2	-	-	0.0	721	2480	6.7	3.2	33.4	
NM	-	-	620	0.0	-	-	0.0	-	-	0.0	239	1524	6.7	3.2	228.6	
NM	-	-	620	2.5	-	-	0.0	-	-	0.0	435	2371	7.6	3.6	122.0	
NM	-	-	620	4.6	-	-	0.0	-	-	0.0	595	2771	8.6	4.1	88.1	

the track.

quation 4) for tests in this table because for each test $\frac{d}{\ell/2}$ had a value of 1.0.

Pass

sic-Variable ediction Term $\frac{G(b\ell)^{3/2}}{W}$	Sand-Track Mobility Number† $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$	Pull Coefficient $\frac{P}{W}$	Torque Coefficient $\frac{M}{Wr}$	Torque Predictor Wr, m - N	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1-S)$	Sinkage Coefficient $\frac{z_R}{\ell}$
18.8	8.8	0.363	0.654	730	0.423	0.123
8.1	5.8	0.267	0.781	1732	0.273	0.171
5.0	4.8	0.274	0.814	3024	0.263	0.155
51.3	24.2	0.484	0.553	737	0.681	0.080
20.2	15.0	0.369	0.571	1831	0.492	0.106
12.8	12.1	0.399	0.653	2979	0.489	0.052
39.5	37.7	0.409	0.599	3032	0.526	0.040
66.3	48.4	0.401	0.602	1772	0.506	0.078
138.3	69.3	0.508	0.551	834	0.710	0.080
13.4	9.6	0.346	0.819	1285	0.314	0.153
18.0	9.8	0.334	0.839	745	0.300	0.132
53.9	16.3	0.550	0.549	304	0.759	0.093
95.0	38.0	0.501	0.566	398	0.666	0.069
50.7	28.5	0.451	0.628	790	0.542	0.085
33.4	23.6	0.329	0.579	1245	0.326	0.055
228.6	96.7	0.566	0.538	445	0.805	0.055
122.0	70.9	0.465	0.517	842	0.678	0.062
88.1	62.1	0.370	0.481	1238	0.601	0.071

3

Test No.	Track Width b,cm	Track Length ℓ,cm	Pen. Resis. Gradient $G, MN/m^3$	Load W,N	Slip S, %	Index of Track Belt Tension t_{tb}, kPa	Pressure (kPa) in Bogie No. 1	2	3	4	5	6	7	1
D-70-0032-1	30.5	61.0	0.65	7,734	16.8	5146	399	364	348	345	-	-	-	8.
33			0.62	13,731	15.0	4265	593	561	548	554	-	-	-	9.
56	30.5	121.9	7.19	12,023	20.6	5372	573	-	554	-	552	-	553	1.
57			7.20	17,703	16.4	4806	653	-	622	-	621	-	621	2.
58			1.18	17,673	20.4	4401	646	-	622	-	621	-	621	1.
59			1.13	11,485	22.9	5033	572	-	553	-	552	-	552	1.
D-70-0035-1	30.5	61.0	0.50	6,871	17.2	5925	525	483	483	483	-	-	-	6.
36			0.60	11,369	16.8	4760	637	616	608	608	-	-	-	7.
37			7.17	11,524	15.6	5398	625	626	621	623	-	-	-	8.
61	30.5	121.9	7.35	6,547	19.5	4155	402	-	347	-	346	-	345	10.
62			7.12	10,982	19.2	4348	603	-	550	-	551	-	551	10
63			7.44	17,785	17.7	3918	626	-	625	-	621	-	622	10
64			0.97	16,991	18.6	4804	666	-	622	-	621	-	621	10
65			1.07	11,251	20.2	4714	601	-	552	-	552	-	552	10.
66			1.06	6,294	22.8	5372	396	-	346	-	552	-	345	8.

* "d" is the distance measured along the base of the track from a point beneath the center line of through the at-rest center of gravity (RCG) of the track when the track rests on an unyielding fl

** Exponent "n" in sand-track mobility number $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2} \cdot \left(\frac{d}{\ell/2}\right)^{n}$ takes a value of 3/2 and a value of 1/2 for $\frac{d}{\ell/2}$ >1.0 (i.e., RCG_h forward of center line).

Table 4

Tests of Single Tracks in Yuma Sand, 20 Percent Slip, First Pass
At-Rest Center of Gravity (RCG) Located Either Forward
or Rearward of Center Line

	) in Bogie No. 5	6	7	Deflection (δ) in Bogie No. 1	2	3	4	5	6	7	Net Torque M, m-N	Pull P, N	Sinkage at Rear Bogie z_R, cm	Trim Angle θ, deg	$\frac{d}{\ell/2}$ *	Basic-variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$
RCG Located Forward of Center Line																
5	-	-	-	8.5	2.9	0.4	0.1	0.0	0.0	0.0	636	2800	4.6	4.3	1.32	6.74
4	-	-	-	9.6	3.1	0.5	0.0	0.0	0.0	0.0	1108	4119	5.9	5.6	1.39	3.62
	552	-	553	1.5	0.0	0.1	0.1	0.0	0.0	0.0	1431	6320	2.5	1.2	1.45	136
	621	-	621	2.0	0.0	0.0	0.0	0.0	0.0	0.0	1995	9156	2.4	1.1	1.55	92.2
	621	-	621	1.5	0.0	0.0	0.0	0.0	0.0	0.0	1710	7222	8.7	4.1	1.55	15.1
	552	-	552	1.3	0.0	0.2	0.0	0.0	0.0	0.0	1132	4735	7.2	3.4	1.45	22.3
RCG Located Rearward of Center Line																
3	-	-	-	6.1	0.0	0.0	0.0	0.0	0.0	0.0	571	1668	16.6	15.8	0.62	5.84
8	-	-	-	7.8	0.0	0.0	0.0	0.0	0.0	0.0	879	1880	15.4	14.6	0.61	4.24
3	-	-	-	8.6	0.0	0.0	0.0	0.0	0.0	0.0	1163	4478	5.6	5.3	0.61	49.9
	346	-	345	10.5	3.9	0.0	0.0	0.0	0.0	0.0	639	3680	5.0	2.4	0.74	255
	551	-	551	10.5	3.9	0.0	0.0	0.0	0.0	0.0	1285	5486	7.6	3.6	0.54	147
	621	-	622	10.5	4.8	0.0	0.0	0.0	0.0	0.0	1667	7299	7.7	3.6	0.44	94.8
	621	-	621	10.5	3.9	0.0	0.0	0.0	0.0	0.0	1319	4067	5.4	2.6	0.44	12.9
	552	-	552	10.5	2.8	0.0	0.0	0.0	0.0	0.0	977	3510	14.4	6.8	0.54	21.6
	552	-	345	8.4	2.4	0.0	0.0	0.0	0.0	0.0	532	2185	12.1	5.7	0.74	38.2

h the center line of the rear road wheel to the vertical line that passes
s on an unyielding flat surface.

takes a value of 3/2 for $\frac{d}{\ell/2} < 1.0$ (i.e., RCG_h rearward of center line),

2

variable iction rm $\frac{(b\ell)^{3/2}}{W}$	Sand-Track Mobility Number** $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2} \cdot \left(\frac{d}{\ell/2}\right)^{n}$	Pull Coefficient $\frac{P}{W}$	Torque Coefficient $\frac{M}{Wr}$	Torque Predictor Wr,m-N	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1-S)$	Sinkage Coefficient $\frac{z_R}{\ell}$
74	7.07	0.362	0.498	1277	0.605	0.075
62	3.74	0.300	0.489	2267	0.521	0.097
	102	0.526	0.721	1985	0.579	0.021
2	81.4	0.516	0.683	2923	0.632	0.020
1	13.3	0.409	0.586	2918	0.556	0.071
3	16.3	0.412	0.597	1896	0.532	0.059
4	1.89	0.243	0.503	1134	0.400	0.272
24	1.52	0.165	0.468	1877	0.293	0.252
[illegible]	18.0	0.389	0.611	1903	0.537	0.092
	93.9	0.562	0.591	1081	0.765	0.041
	34.6	0.500	0.709	1813	0.570	0.062
[illegible]	19.7	0.410	0.568	2936	0.594	0.063
[illegible]	2.62	0.239	0.470	2756	0.414	0.044
[illegible]	5.16	0.312	0.526	1858	0.473	0.118
2	13.8	0.347	0.512	1039	0.523	0.099

Test No.	Track Width b cm	Track Length ℓ cm	Pen. Resist. Gradient G, MN/m^3	Load W, N	Slip S, %	Index of Track Belt Tension t_{tb}, kPa	Pressure (kPa) in Bogie No.* 1	2	3	4	5	6
D-71-0031-1	15.2	61.0	1.12	4,687	16.2	2051	620	620	620	620	-	-
32			1.13	11,258	19.3	2218	617	620	621	621	-	-
33			1.17	17,904	26.9	118	608	616	620	622	-	-
1	61.0	61.0	0.97	4,910	20.5	3265	678	621	623	-	-	-
2			0.99	11,387	19.6	1390	664	632	621	-	-	-
3			1.03	18,499	21.1	1359	669	662	618	-	-	-
55	15.2	121.9	0.98	5,261	11.2	1190	620	-	620	-	620	-
56			1.06	10,064	9.3	1063	620	-	620	-	620	-
57			1.04	14,971	9.3	1152	619	-	620	-	620	-
88	61.0	121.9	0.99	25,487	17.8	1198	615	-	619	-	618	-
89			1.11	14,969	26.0	1379	619	-	619	-	618	-
90			1.12	5,139	21.4	1384	620	-	620	-	619	-
D-71-0034-1	15.2	61.0	0.97	17,796	14.7	3490	614	619	620	621	-	-
35			0.97	11,104	20.5	4758	614	620	620	620	-	-
36			1.03	4,394	17.7	5492	615	620	620	620	-	-
4	61.0	61.0	1.12	18,479	19.6	3371	642	648	618	-	-	-
5			1.15	11,435	21.3	3838	634	642	617	-	-	-
6			1.10	4,896	20.8	4121	642	620	615	-	-	-
D-71-0058-1	15.2	121.9	1.12	15,183	22.4	4134	621	-	620	-	621	-
59			1.08	10,077	20.9	4134	620	-	620	-	620	-
60			1.11	5,227	24.9	4135	620	-	620	-	620	-
85	61.0	121.9	0.99	5,009	22.8	4108	620	-	620	-	619	-
86			1.07	14,956	23.0	4118	619	-	619	-	619	-
87			0.98	25,493	22.6	2129	616	-	619	-	619	-

* Road bogies are numbered consecutively starting at the rear of the track.

** Here, trim angle θ is defined as $\theta = \sin^{-1}(z_R/\ell)$.

† $\left(\frac{d}{\ell/2}\right)^n$ is not included as part of the sand-track mobility number (equation 4) for tests in

Table 5

Tests of Single Tracks in Yuma Sand, 20 Percent Slip, First Pass
Index of Track Belt Tension Initially Set at 1380- and 4140-kPa

(kPa) in Bogie No.*					Deflection (δ) in cm of Bogie No.							Net Torque	Pull	Sinkage at Rear Bogie	Trim** Angle	Basic Vari[a]... Prediction
3	4	5	6	7	1	2	3	4	5	6	7	M,m-N	P, N	z_R, cm	θ, deg	$\frac{G(b\ell)^{3/2}}{W}$
								Index of Track Belt Tension Initially Set at 1380 kPa								
620	620	-	-	-	0.0	0.0	0.0	0.0	-	-	-	578	846	16.7	15.9	6.6
621	621	-	-	-	3.7	0.0	0.0	0.0	-	-	-	[illegible]	1793	21.3	20.4	2.8
620	622	-	-	-	6.6	2.1	0.0	0.0	-	-	-	[illegible]	1641	20.8	19.9	1.8
623	-	-	-	-	1.8	0.0	0.0	0.0	-	-	-	[illegible]	1400	10.5	9.9	44.8
621	-	-	-	-	4.9	0.4	0.0	0.0	-	-	-	1085	3980	10.4	9.8	19.7
618	-	-	-	-	7.6	2.6	0.0	0.0	-	-	-	1482	5773	7.3	6.9	12.6
620	-	620	-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	607	1913	17.4	8.2	14.6
620	-	620	-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1192	2750	22.3	10.5	8.2
620	-	620	-	620	0.6	0.0	0.0	0.0	0.0	0.0	0.0	1788	3633	26.3	12.5	5.4
619	-	618	-	620	9.4	7.3	4.4	1.5	0.0	0.0	0.0	2428	10893	8.8	4.1	24.9
619	-	618	-	620	1.3	0.8	0.3	0.0	0.0	0.0	0.0	1411	5773	15.4	7.2	47.5
620	-	619	-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	602	2759	7.9	3.7	139.2
								Index of Track Belt Tension Initially Set at 4140 kPa								
620	621	-	-	-	6.9	2.2	0.0	0.0	-	-	-	2127	2658	20.9	20.1	1.5
620	620	-	-	-	3.9	0.0	0.0	0.0	-	-	-	1586	2115	22.0	21.2	2.4
620	620	-	-	-	0.2	0.0	0.0	0.0	-	-	-	754	1211	16.0	15.2	6.5
618	-	-	-	-	7.6	2.6	0.0	0.1	-	-	-	1544	6289	6.4	6.0	13.7
617	-	-	-	-	5.1	0.4	0.0	0.0	-	-	-	1016	4323	9.3	8.7	22.8
615	-	-	-	-	2.1	0.0	0.0	0.0	-	-	-	648	2032	9.5	9.0	51.0
620	-	621	-	621	1.6	0.4	0.0	0.0	0.0	0.0	0.0	1710	3261	25.3	12.0	5.8
620	-	620	-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1276	2680	21.7	10.3	8.4
620	-	620	-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	647	1938	16.3	7.7	16.6
620	-	619	-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	548	2603	6.0	2.8	126.7
619	-	619	-	620	1.0	0.4	0.1	0.0	0.0	0.0	0.0	1331	5396	11.3	5.3	45.9
619	-	619	-	620	9.3	7.1	4.2	1.4	0.0	0.0	0.0	2438	10890	11.5	5.4	24.6

track.

quation 4) for tests in this table because for each test $\frac{d}{\ell/2}$ had a value of 1.0.

2

Pass
kPa

Basic Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$	Sand-Track Mobility Number † $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$	Pull Coefficient $\frac{P}{W}$	Torque Coefficient $\frac{M}{Wr}$	Torque Predictor Wr, m-N	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1-S)$	Sinkage Coefficient $\frac{z_p}{\ell}$
6.6	3.2	0.180	0.747	774	0.202	0.274
2.8	2.1	0.159	0.757	1859	0.170	0.349
1.8	1.7	0.092	0.648	2956	0.104	0.341
44.8	22.1	0.285	0.498	811	0.455	0.172
19.7	14.8	0.350	0.577	1880	0.488	0.170
12.6	12.1	0.312	0.485	3054	0.508	0.120
14.6	5.6	0.364	0.699	869	0.462	0.143
8.2	4.4	0.273	0.717	1662	0.345	0.183
5.4	3.5	0.243	0.723	2472	0.305	0.216
24.9	21.2	0.427	0.577	4208	0.608	0.072
47.5	31.0	0.386	0.571	2471	0.500	0.126
139.2	53.2	0.535	0.710	852	0.592	0.065
1.5	1.4	0.149	0.724	2938	0.176	0.343
2.4	1.8	0.190	0.865	1833	0.175	0.361
6.5	3.0	0.276	1.039	725	0.219	0.262
13.7	13.1	0.340	0.506	3051	0.540	0.105
22.8	17.2	0.378	0.538	1888	0.553	0.152
51,0	25.2	0.415	0.802	808	0.410	0.156
5.8	3.8	0.215	0.682	2507	0.245	0.208
8.4	4.5	0.266	0.767	1664	0.274	0.178
16.6	6.4	0.371	0.750	863	0.371	0.134
126.7	47.8	0.520	0.663	827	0.605	0.049
45.9	29.9	0.361	0.539	2469	0.516	0.093
24.6	20.9	0.427	0.579	4209	0.571	0.094

3

Test No.	Track Width b cm	Track Length ℓ cm	Pen. Resist. Gradient $G, MN/m^3$	Load W, N	Slip S, %	Index of Track Belt Tension t_{tb}, kPa	Pressure (kPa) in Bogie No.* 1	2	3	4	5	6	7
D-71-0025-1	15.2	61.0	1.15	5,062	15.9	7791	618	445	274	115	-	-	-
26			1.10	11,471	17.8	5472	620	445	275	84	-	-	-
27			1.05	14,872	19.7	1271	618	445	274	100	-	-	-
7	61.0	61.0	1.12	5,197	13.8	6205	619	447	275	98	-	-	-
8			1.13	11,869	19.2	2617	619	446	275	98	-	-	-
9			1.12	18,551	22.3	1189	619	446	275	102	-	-	-
61	15.2	121.9	1.00	5,148	23.6	6889	620	-	448	-	277	-	103
62			1.15	10,256	23.0	6898	621	-	448	-	277	-	105
63			1.10	15,438	23.5	6905	620	-	448	-	277	-	103
91	61.0	121.9	0.81	5,537	23.1	6725	619	-	448	-	274	-	102
92			1.07	16,139	23.1	3871	620	-	448	-	274	-	102
93			0.77	26,284	21.0	646	613	-	446	-	274	-	103
D-71-0028-1	15.2	61.0	1.19	18,566	23.9	4616	107	279	448	620	-	-	-
29			1.17	11,655	16.5	6903	102	274	448	621	-	-	-
30			1.33	3,589	22.9	5456	104	277	449	621	-	-	-
10	61.0	61.0	0.99	18,654	19.5	3627	103	275	447	619	-	-	-
11			0.96	11,359	22.3	5148	103	274	447	615	-	-	-
12			1.10	4,006	22.6	6078	100	272	447	620	-	-	-
D-71-0064-1	15.2	121.9	1.12	15,209	13.6	6901	104	-	276	-	448	-	622
65			1.12	9,991	14.6	6904	102	-	276	-	448	-	621
66			1.11	5,395	8.7	6906	103	-	276	-	448	-	621
94	61.0	121.9	0.97	25,906	2.7	3379	102	-	277	-	445	-	620
95			0.72	15,027	4.0	4019	102	-	275	-	445	-	621
96			0.69	5,459	5.5	4867	101	-	275	-	444	-	620

* Road bogies are numbered consecutively starting at the rear end of the track.

** Here, trim angle θ is defined as $\theta = \sin^{-1} (z_R / \ell)$.

† $\left(\frac{d}{\ell/2}\right)^n$ is not included as part of the sand-track mobility number (equation 4) for tests in thi

Table 6

Tests of Single Tracks in Yuma Sand, 20 Percent Slip, First Pass
Uniform Change in Road-Bogie Cylinder Pressure over Length of Track

) in Bogie No.* 4	5	6	7	Deflection (δ) in cm of Bogie No. 1	2	3	4	5	6	7	Net Torque M, m-N	Pull P, N	Sinkage at Rear Bogie z_R, cm	Trim** Angle θ, deg	Basic-Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$
Pressure in Road-Bogie Cylinders Uniformly Increasing Front-to-Rear															
115	-	-	-	2.2	0.0	0.0		-	-	-	613	1,395	14.8	14.0	6.3
84	-	-	-	7.6	5.8	6.5	0.4	-	-	-	1391	2,501	10.7	10.1	2.7
100	-	-	-	10.4	8.9	10.6	6.3	-	-	-	1752	2,857	6.8	6.4	2.0
98	-	-	-	3.0	0.0	0.0	0.0	-	-	-	409	2,366	8.4	7.9	48.9
98	-	-	-	8.2	4.9	3.4	0.6	-	-	-	1102	5,044	6.0	5.6	21.9
102	-	-	-	10.5	9.8	10.8	7.0	-	-	-	1695	6,570	0.5	0.5	13.7
-	277	-	103	0.0	0.0	0.0	0.0	0.0	0.0	0.0	637	1,898	17.0	8.0	15.2
-	277	-	105	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1223	2,970	23.4	11.1	8.8
-	277	-	103	3.7	2.1	0.7	0.0	0.0	0.0	0.0	1844	4,136	26.5	12.5	5.6
-	274	-	102	0.0	0.0	0.0	0.0	0.0	0.0	0.3	479	2,706	8.5	4.0	93.8
-	274	-	102	5.5	4.9	3.2	1.1	0.0	0.0	0.0	1580	6,498	13.0	6.1	42.5
-	274	-	103	10.4	10.8	10.8	9.5	9.7	9.6	7.0	2480	10,647	8.7	4.1	18.8
Pressure in Road-Bogie Cylinders Uniformly Decreasing Front-to-Rear															
620	-	-	-	7.3	8.2	5.7	3.3	-	-	-	2318	3,274	14.3	13.5	1.8
621	-	-	-	7.1	5.4	1.7	0.2	-	-	-	1555	2,998	11.4	10.8	2.8
621	-	-	-	3.3	1.5	0.0	0.0	-	-	-	461	1,676	6.6	6.2	10.3
619	-	-	-	7.3	7.3	4.5	1.5	-	-	-	1533	6,499	1.1	1.0	12.0
615	-	-	-	7.2	4.8	1.7	-0.1	-	-	-	1044	4,470	0.7	0.6	19.8
620	-	-	-	4.0	1.5	0.0	-0.1	-	-	-	408	2,222	8.4	7.9	62.3
-	448	-	622	10.3	7.7	3.8	0.7	0.0	0.0	0.0	1897	5,019	10.5	4.9	5.8
-	448	-	621	10.0	5.9	2.0	0.0	0.0	0.0	0.0	1413	4,108	17.0	8.0	8.8
-	448	-	621	6.3	3.1	0.0	0.0	0.0	0.0	0.0	721	2,550	15.6	7.4	16.1
-	445	-	620	7.5	10.4	7.4	3.9	1.8	0.0	0.0	2411	10,488	2.9	1.3	24.0
-	445	-	621	7.5	8.5	5.1	1.7	0.0	0.0	0.0	1634	7,613	1.6	0.7	30.7
-	444	-	620	5.2	5.2	1.8	0.0	0.0	0.0	0.0	584	3,417	9.4	4.4	81.0

track.

ion 4) for tests in this table because for each test $\frac{d}{\ell/2}$ had a value of 1.0.

ss
Track

Basic-Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$	Sand-Track Mobility Number † $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$	Pull Coefficient $\frac{P}{W}$	Torque Coefficient $\frac{M}{Wr}$	Torque Predictor Wr, m-N	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1-S)$	Sinkage Coefficient $\frac{z_R}{\ell}$
ear						
6.3	3.2	0.276	0.733	836	0.317	0.242
2.7	2.3	0.218	0.734	1894	0.244	0.175
2.0	1.7	0.192	0.714	2455	0.216	0.111
48.9	24.9	0.455	0.477	858	0.822	0.138
21.9	16.8	0.432	0.562	1930	0.621	0.098
13.7	13.2	0.354	0.665	3062	0.414	0.008
15.2	5.8	0.369	0.749	850	0.376	0.139
8.8	4.7	0.290	0.722	1693	0.309	0.192
5.6	3.7	0.268	0.723	2549	0.284	0.217
93.8	37.2	0.489	0.524	914	0.718	0.070
42.5	28.8	0.403	0.593	2664	0.523	0.107
18.8	16.2	0.405	0.571	4339	0.560	0.071
ear						
1.8	1.8	0.176	0.756	3065	0.177	0.234
2.8	2.8	0.257	0.808	1924	0.266	0.187
10.3	10.3	0.467	0.778	593	0.463	0.108
12.0	12.0	0.348	0.498	3080	0.563	0.018
19.8	19.8	0.394	0.557	1875	0.550	0.011
62.3	62.3	0.555	0.617	662	0.696	0.138
5.8	5.8	0.330	0.755	2511	0.378	0.086
8.8	8.8	0.411	0.857	1650	0.410	0.139
16.1	15.5	0.473	0.809	891	0.534	0.128
24.0	24.0	0.404	0.564	4277	0.697	0.023
30.7	30.7	0.507	0.659	2481	0.739	0.013
81.0	78.3	0.626	0.648	901	0.913	0.077

3

Test No.	Track Width b cm	Track Length ℓ cm	Pen. Resis. Gradient $G, MN/m^3$	Load W, N	Slip S, %	Index of Track Belt Tension t_{tb}, kPa	Pressure (kPa) in Bogie No.* 1	2	3	4	5	6	7	D… 1
D-71-0037-1	15.2	61.0	0.99	4,659	14.1	10,306	620	620	622	621	-	-	-	1.1
38			1.09	10,967	17.9	11,426	620	620	622	616	-	-	-	3.7
39			1.15	17,889	17.7	12,257	620	620	617	616	-	-	-	7.3
40	30.5	61.0	1.19	18,358	22.5	9,884	620	620	618	617	-	-	-	2.8
41			1.21	10,803	21.4	8,110	619	620	617	617	-	-	-	4.6
42			1.15	4,691	17.7	7,373	620	620	620	614	-	-	-	1.5
43	61.0	61.0	1.01	5,085	17.8	7,348	619	620	620	616	-	-	-	2.1
44			1.08	11,176	22.3	9,230	620	620	622	621	-	-	-	4.9
45			1.04	19,157	19.4	9,902	620	620	620	616	-	-	-	7.8
52	15.2	121.9	1.06	5,268	27.5	6,892	620	-	620	-	620	-	620	0.0
53			1.08	15,087	19.3	8,503	620	-	620	-	620	-	618	0.7
54			1.19	10,314	26.7	6,897	620	-	620	-	621	-	620	0.0
49	30.5	121.9	1.09	5,157	20.5	7,994	620	-	620	-	620	-	620	0.0
50			1.07	15,000	21.6	9,998	621	-	621	-	622	-	622	0.0
46	61.0	121.9	1.13	25,601	22.1	7,511	620	-	620	-	620	-	620	8.
47			1.10	15,519	22.8	7,438	621	-	620	-	621	-	621	0.
48			1.09	5,302	21.5	7,523	620	-	620	-	620	-	620	0.

* Road bogies are numbered consecutively starting at the rear end of the track.

** Here, trim angle θ is defined as $\theta = \sin^{-1}(z_R/\ell)$.

† $\left(\frac{d}{\ell/2}\right)^n$ is not included as part of the sand-track mobility number (equation 4) for tests in thi

Table 7

Tests of Single Tracks in Yuma Sand, 20 Percent Slip, First Pass
Tracks Powered by Front-Sprocket Drive

No.* 6	7	Deflection (δ) in cm. of Bogie No 1	2	3	4	5	6	7	Net Torque M, m-N	Pull P, N	Sinkage at Rear Bogie z_R, cm	Trim** Angle θ, deg	Basic-Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$	Sand-Tra[illegible] Mobility Number † $\frac{G(b\ell)^{3/2}}{W}$.
-	-	1.1	0.0	0.0	0.0	-	-	-	628	1,109	18.1	17.3	5.9	2[illegible]
-	-	3.7	0.0	0.0	0.0	-	-	-	1365	1,482	23.0	22.2	2.8	2[illegible]
-	-	7.3	3.0	0.0	0.0	-	-	-	2183	1,887	20.8	19.9	1.8	1[illegible]
-	-	2.8	1.8	0.0	0.0	-	-	-	1502	4,834	11.2	10.6	5.2	5[illegible]
-	-	4.6	0.8	0.0	0.0	-	-	-	965	3,409	11.6	11.0	9.0	6[illegible]
-	-	1.5	0.0	0.0	0.0	-	-	-	450	1,557	10.0	9.6	19.7	9[illegible]
-	-	2.1	0.0	0.0	0.0	-	-	-	373	2,190	10.4	9.8	45.0	22[illegible]
-	-	4.9	1.0	0.0	0.0	-	-	-	985	4,531	8.9	8.4	21.9	16[illegible]
-	-	7.8	4.4	0.0	0.0	-	-	-	1554	6,632	5.1	4.8	12.3	12[illegible]
-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	579	1,842	9.0	4.2	15.7	[illegible]
-	618	0.7	0.0	0.0	0.0	0.0	0.0	0.7	1714	2,889	14.8	7.0	5.6	[illegible]
-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1194	2,658	22.4	10.6	9.0	[illegible]
-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	491	2,480	5.3	2.5	47.9	1[illegible]
-	622	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1291	4,299	8.7	4.1	18.1	1[illegible]
-	620	8.1	6.5	4.2	1.6	0.0	0.0	0.0	2473	10,955	8.9	4.2	28.3	2[illegible]
-	621	0.0	0.1	0.2	0.0	0.0	0.0	0.0	1448	6,097	9.5	4.5	45.4	3[illegible]
-	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	535	2,869	7.0	3.6	131.8	[illegible]

[illegible]or tests in this table because for each test $\frac{d}{\ell/2}$ had a value of 1.0.

2

Basic-Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$	Sand-Track Mobility Number † $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$	Pull Coefficient $\frac{P}{W}$	Torque Coefficient $\frac{M}{Wr}$	Torque Predictor Wr, m-N	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1-S)$	Sinkage Coefficient $\frac{z_R}{\ell}$
5.9	2.8	0.238	0.816	769	0.251	0.297
2.8	2.1	0.135	0.754	1811	0.147	0.378
1.8	1.7	0.105	0.739	2953	0.117	0.341
5.2	5.0	0.263	0.496	3031	0.411	0.184
9.0	6.6	0.316	0.541	1784	0.459	0.190
19.7	9.5	0.332	0.581	774	0.470	0.164
45.0	22.6	0.431	0.444	840	0.798	0.170
21.9	16.3	0.405	0.534	1845	0.589	0.146
12.3	12.0	0.346	0.491	3163	0.568	0.084
15.7	6.1	0.350	0.666	870	0.381	0.074
5.6	3.7	0.191	0.688	2491	0.224	0.121
9.0	4.9	0.258	0.701	1703	0.270	0.184
47.9	18.3	0.481	0.577	851	0.663	0.043
18.1	11.8	0.287	0.521	2477	0.487	0.071
28.3	24.1	0.428	0.585	4227	0.570	0.073
45.4	30.1	0.393	0.565	2562	0.537	0.078
131.8	51.1	0.541	0.611	875	0.695	0.057

3

Test No.	Sand Type	Track Width b, cm	Track Length ℓ, cm	Before-Traffic Penetration Resistance Gradient G, MN/m^3	Initial Settings: Index of Track Belt Tension t_{tb}, kPa	Initial Settings: Pressure in Road Bogies p_b, kPa	At Towed Point: Load W, N	At Towed Point: Slip S, %	At Towed Point: Towed Force P_T, N	At Towed Point: Towed Force Coefficient P_T/W	At Towed Point: Sinkage z_F, cm	At Towed Point: Sinkage Coefficient z_F/ℓ	At Towed Point: N_s*	Load W, N
4-67-0001-A	Yuma	15.2	121.9	0.73	3450	276	3,492	-0.9	548	0.157	3.15	0.026	7.89	3,426
2				0.94			6,796	-1.5	890	0.131	4.04	0.033	7.25	6,806
3				1.52	1380	103	3,394	-1.8	533	0.157	1.64	0.013	27.2	3,301
4				1.59			3,425	-0.8	401	0.117	2.24	0.018	27.0	3,416
5				1.33		103-186**	3,505	-0.5	452	0.129	2.18	0.018	17.5	3,492
6				1.47		186-103†	3,893	-0.6	580	0.149	2.07	0.017	24.6	3,447
7				1.64		103	3,496	-0.7	535	0.153	2.02	0.017	28.9	3,447
8				2.77		241	6,761	-0.6	629	0.093	1.97	0.016	23.0	6,748
9		30.5	121.9	1.47	4140		6,780	-0.6	1140	0.168	1.29	0.011	34.7	6,868
10				1.33			7,077	-0.8	934	0.132	1.12	0.009	30.7	7,064
32				4.53	6890	276	2,055	-0.5	382	0.186	1.63	0.013	181	2,126
33				4.48			2,077	-0.3	413	0.199	1.55	0.013	178	2,082
34				4.23			6,784	-1.0	1248	0.184	1.47	0.012	92.9	6,770
35				4.72			6,850	-1.0	980	0.143	1.42	0.012	103	6,828
36				4.36			4,426	-0.8	677	0.153	1.83	0.015	119	4,439
37				4.42			4,381	-0.3	832	0.190	1.78	0.015	121	4,381
43				3.04			7,006	-0.5	1016	0.145	1.70	0.014	65.9	6,748
44				2.59			6,539	-1.0	1216	0.186	1.22	0.010	58.0	6,552
46				2.91			4,390	-0.3	549	0.125	1.37	0.011	79.6	4,417
A-68-0002-1				2.55			2,077	-1.0	418	0.201	1.19	0.010	99.5	2,091
4				3.75			2,135	-1.0	320	0.150	1.35	0.011	147	2,157
6				3.47			4,551	-1.0	733	0.161	1.37	0.011	93.3	4,551
8				3.57			6,841	-0.5	636	0.093	1.09	0.009	78.2	6,828
9				1.12			6,659	-1.5	1292	0.194	2.51	0.021	24.9	6,672
12				1.02			4,350	-1.8	1022	0.235	2.54	0.021	28.0	4,350
13				2.23			4,359	-1.3	1055	0.242	1.37	0.011	61.3	4,359
15				1.11			2,215	-1.5	356	0.097	1.96	0.016	43.0	2,215
A-68-0018-1				2.24			6,508	-0.8	1002	0.154	1.57	0.013	50[illegible]	6,494
20				1.74			2,171	-0.7	458	0.211	1.96	0.016	67.9	2,184
D-69-0164-1				2.00			13,012	-0.4	1653	0.127	0.71	0.006	31.7	13,012
190				0.94			9,861	-0.5	1193	0.121	3.39	0.028	17.1	9,740
191				1.06			12,927	-0.1	1422	0.110	1.34	0.011	16.9	12,927
192				4.53			13,820	-0.8	1852	0.134	0.28	0.002	69.8	13,884
193				4.36			9,959	-0.7	1165	0.117	2.86	0.023	79.1	9,964
194				3.39			10,420	-0.4	1386	0.133	3.63	0.030	60.2	10,28
195				3.46			11,737	-0.3	1291	0.110	2.64	0.022	57.9	11,65
D-70-0021-1		61.0	61.0	1.99			14,013	-0.1	1668	0.119	2.08	0.034	32.2	13,40
22				2.02		483	26,560	-0.7	2975	0.112	4.52	0.074	17.3	27,18
26		30.5	61.0	4.89		276	13,240	-0.4	1496	0.113	1.95	0.032	29.6	13,86
27				4.73		483	26,565	-1.4	3241	0.122	4.37	0.072	14.3	27,15
D-71-0001-2	Mortar	15.2	121.9	0.90		621	15,679	-6.3	5B17	0.371	17.08	0.140	3.06	15,679
2				0.91			11,081	-3.7	3213	0.290	13.10	0.106	3.68	11,252
3				0.95			6,230	-0.3	1682	0.270	8.45	0.069	5.13	6,417
4				4.26			5,120	-0.3	543	0.106	4.09	0.034	25.3	5,592
5				3.87			10,401	-0.3	1134	0.109	5.23	0.043	16.1	10,724
6				4.01			15,429	-1.3	2299	0.149	5.37	0.044	13.7	15,299
94		30.5	61.0	1.15			16,042	-2.5	2904	0.181	3.33	0.055	5.13	16,473
95				1.14			10,606	-2.7	1602	0.151	6.72	0.110	6.26	10,357
96				1.03			5,297	-1.5	620	0.117	5.76	0.094	8.00	5,89

* N_s = sand - track mobility number = $\frac{G\,(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2} \cdot \left(\frac{d}{\ell/2}\right)^{n}$; for all tests reported in this table, $\frac{d}{\ell/2} = 1.0$.

** Values of p_b increased from front to rear of track.

† Values of p_b decreased from front to rear of track.

Table 8

Programmed-Increasing-Slip Tests in Yuma and in Mortar Sands

			At Self-Propelled Point								At Maximum-Tractive-Efficiency Point						
kage cm	Sinkage Coefficient z_F/ℓ	N_s*	Load W, N	Slip S, %	Net Torque M, m - N	Torque Predictors W · r, m - N	Sinkage z_R, cm	Sinkage Coefficient z_R/ℓ	N_s	Load W, N	Slip S, %	Net Torque M, m - N	Torque Predictors W · r, m - N	Pull P, N	Pull Coefficient P/W	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1 - S)$
15	0.026	7.89	3,426	0.8	12	566	3.11	0.026	7.95	3,604	15.7	312	596	1500	0.416	0.670
04	0.033	7.25	6,806	1.4	102	1124	4.50	0.037	7.25	7,082	9.3	461	1170	2026	0.286	0.658
64	0.013	27.2	3,301	1.7	62	545	1.94	0.016	27.6	3,452	16.1	371	570	1788	0.518	0.668
24	0.018	27.0	3,416	0.7	49	564	2.18	0.018	28.4	3,381	12.6	300	558	1491	0.441	0.717
18	0.018	17.5	3,492	0.5	37	577	2.18	0.018	17.5	3,350	11.2	252	553	1290	0.385	0.750
07	0.017	24.6	3,447	0.6	33	569	2.18	0.018	26.1	3,505	14.6	317	579	1633	0.466	0.727
02	0.017	28.9	3,447	0.6	45	569	2.12	0.017	29.1	3,541	9.9	305	585	1593	0.450	0.778
97	0.016	23.0	6,748	0.6	54	1114	1.97	0.016	23.0	6,670	9.5	506	1118	2518	0.372	0.744
29	0.011	34.7	6,868	0.5	119	1134	1.98	0.016	34.4	7,464	13.1	774	1232	3866	0.518	0.717
12	0.009	30.7	7,064	0.8	84	1166	1.71	0.014	30.7	7,255	12.3	785	1198	3511	0.484	0.648
63	0.013	181	2,126	0.6	34	351	2.03	0.017	178	2,260	5.2	213	373	1171	0.518	0.860
55	0.013	178	2,082	0.2	20	344	1.68	0.014	178	2,197	8.1	224	363	1265	0.576	0.858
47	0.012	92.9	6,770	1.0	68	1118	1.68	0.014	93.4	6,806	7.3	691	1124	3587	0.527	0.795
42	0.012	103	6,828	1.0	104	1127	1.55	0.013	104	6,770	8.3	647	1118	3818	0.504	0.799
83	0.015	119	4,439	0.7	39	733	1.96	0.016	118	4,470	7.3	417	738	2476	0.554	0.909
78	0.015	121	4,381	0.3	28	723	1.96	0.016	121	4,479	6.0	422	739	2365	0.528	0.869
70	0.014	65.9	6,748	0.5	84	1114	1.93	0.016	67.0	6,815	6.1	625	1125	3421	0.502	0.848
22	0.010	58.0	6,552	1.0	134	1082	1.57	0.013	58.0	6,517	8.7	677	1076	3556	0.515	0.747
37	0.011	79.6	4.417	0.2	22	729	1.65	0.014	79.3	4,493	7.4	418	742	2462	0.548	0.901
19	0.010	99.5	2,091	1.0	57	345	1.47	0.012	101	2,148	6.1	222	355	1209	0.563	0.845
35	0.011	147	2,157	1.0	42	356	1.57	0.013	146	2,237	6.0	238	369	1300	0.581	0.847
37	0.011	93.3	4,551	1.0	57	751	1.47	0.012	93.3	4,573	5.5	463	755	2945	0.594	0.915
09	0.009	78.2	6,828	0.5	60	1127	1.35	0.011	78.3	6,841	6.1	629	1129	3708	0.542	0.913
51	0.021	24.9	6,672	1.5	130	1102	2.69	0.022	24.8	6,463	9.3	620	1067	2844	0.440	0.687
54	0.021	28.0	4,350	1.7	11	718	2.87	0.024	28.0	4,359	9.1	366	720	2036	0.467	0.835
37	0.011	61.3	4,359	1.2	10	720	1.80	0.015	61.2	4,373	9.4	420	722	2493	0.570	0.888
96	0.016	43.0	2,215	1.5	47	366	2.57	0.021	42.9	2,215	6.5	217	366	1125	0.508	0.801
57	0.013	50.3	6,494	0.8	88	1072	1.88	0.015	50.4	6,450	6.6	603	1065	2980	0.462	0.762
96	0.016	67.9	2,184	0.8	28	361	2.62	0.021	67.6	2,220	6.4	221	367	1134	0.511	0.794
71	0.006	31.7	13,012	0.8	230	2148	2.97	0.024	31.7	13,609	7.7	1173	2247	5035	0.370	0.654
39	0.028	17.1	9,740	0.8	104	1608	6.28	0.052	17.3	8,914	8.8	663	1472	3405	0.382	0.773
34	0.011	16.9	12,927	0.3	137	2134	2.64	0.022	16.9	13,892	10.0	1241	2294	6835	0.492	0.819
28	0.002	69.8	13,884	1.1	369	2292	2.54	0.021	69.7	13,239	9.8	1353	2186	7652	0.578	0.842
86	0.023	79.1	9,964	1.9	134	1645	4.02	0.033	79.2	8,728	5.1	710	1441	3954	0.453	0.873
63	0.030	60.2	10,282	0.6	156	1698	4.17	0.034	60.6	9,072	4.9	733	1498	4173	0.460	0.894
64	0.022	57.9	11,650	0.5	144	1923	2.10	0.017	58.1	11,863	7.2	1235	1959	6703	0.565	0.832
08	0.034	32.2	13,405	1.7	217	2213	2.12	0.035	33.7	12,680	13.9	1008	2093	4818	0.380	0.679
52	0.074	17.3	27,185	0.7	458	4488	5.22	0.086	16.9	27,438	7.9	1988	4530	8945	0.326	0.684
95	0.032	29.6	13,865	0.8	254	2289	2.22	0.036	28.3	13,845	6.9	1018	2286	5053	0.365	0.763
37	0.072	14.3	27,153	1.6	601	4483	4.65	0.076	14.0	28,246	10.0	2155	4663	9773	0.346	0.674
08	0.140	3.06	15,679	4.1	331	2589	16.98	0.139	3.06	15,362	16.0	1750	2536	3118	0.203	0.247
10	0.106	3.68	11,252	2.0	176	1858	12.09	0.099	3.65	10,263	12.0	728	1694	1909	0.186	0.381
45	0.069	5.13	6,417	1.6	150	1059	8.50	0.070	5.08	5,504	9.0	361	909	1222	0.222	0.509
09	0.034	25.3	5,592	0.5	73	923	4.27	0.035	24.2	5,278	7.2	385	871	2111	0.400	0.840
23	0.043	16.1	10,724	0.6	237	1771	4.95	0.041	16.0	10,745	8.9	665	1774	3138	0.292	0.710
37	0.044	13.7	15,299	0.9	350	2526	5.65	0.046	13.8	15,140	8.0	1062	2500	3785	0.250	0.541
33	0.055	5.13	16,473	5.4	411	2720	5.61	0.092	5.07	15,134	20.0	1328	2499	4419	0.292	0.440
72	0.110	6.26	10,357	6.3	209	1710	8.15	0.134	6.34	10,243	16.0	677	1691	2356	0.230	0.483
76	0.094	8.00	5,893	0.6	84	973	5.59	0.092	7.58	5,645	22.0	620	932	1981	0.351	0.412

e, $\frac{d}{\ell/2} = 1.0$.

...ive-Efficiency Point						At 20-Percent Slip Point								
...ll ..., N	Pull Coefficient P/W	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1 - S)$	Sinkage z_R, cm	Sinkage Coefficient z_R/ℓ	N_s	Load W, N	Net Torque M, m - N	Torque Predictor W • r, m - N	Pull P, N	Pull Coefficient P/W	Tractive Efficiency $\frac{P/W}{M/Wr} \cdot (1 - S)$	Sinkage z_R, cm	Sinkage Coefficient z_R/ℓ	N_s
00	0.416	0.670	8.43	0.069	7.77	3,465	379	572	1,512	0.436	0.526	9.58	0.079	7.91
26	0.286	0.658	9.93	0.081	7.13	7,188	717	1187	2,909	0.405	0.536	17.02	0.140	7.05
88	0.518	0.668	9.28	0.076	27.0	3,531	431	583	1,957	0.554	0.600	10.60	0.087	26.7
91	0.441	0.717	8.19	0.067	28.5	3,567	395	589	1,779	0.499	0.595	9.82	0.081	27.8
90	0.385	0.750	6.06	0.050	17.8	3,514	328	580	1,512	0.430	0.608	8.29	0.068	17.4
33	0.466	0.727	8.19	0.067	25.9	3,630	359	599	1,779	0.490	0.654	8.84	0.073	25.5
93	0.450	0.778	8.08	0.066	28.7	3,750	417	619	1,997	0.533	0.633	10.86	0.089	28.0
18	0.372	0.744	8.92	0.073	22.9	7,366	737	1216	3,114	0.423	0.558	12.41	0.102	22.0
66	0.518	0.717	7.94	0.065	33.0	7,638	834	1261	4,071	0.533	0.645	9.25	0.076	32.6
11	0.484	0.648	8.76	0.072	30.3	7,611	799	1257	3,897	0.512	0.644	9.78	0.080	29.6
71	0.518	0.860	2.11	0.017	173	2,246	242	371	1,253	0.558	0.684	3.25	0.027	173
65	0.576	0.858	1.91	0.016	173	2,189	239	361	1,296	0.592	0.715	2.72	0.022	174
87	0.527	0.795	1.83	0.015	93.0	6,784	701	1120	3,870	0.571	0.730	3.91	0.032	93.1
18	0.504	0.799	1.68	0.014	103	6,761	728	1109	4,012	0.593	0.723	3.38	0.028	104
76	0.554	0.909	2.09	0.017	119	4,439	480	731	2,891	0.651	0.793	3.18	0.026	119
65	0.528	0.869	1.98	0.016	120	4,404	475	727	2,713	0.616	0.754	3.48	0.029	121
21	0.502	0.848	2.20	0.018	66.7	6,744	677	1114	3,470	0.515	0.688	4.90	0.040	67.1
56	0.515	0.747	2.49	0.020	57.7	6,494	721	1054	3,381	0.521	0.609	5.08	0.042	58.2
62	0.548	0.901	1.83	0.015	78.2	4,448	440	734	2,624	0.590	0.787	3.86	0.032	79.1
09	0.563	0.845	1.60	0.013	99.6	2,197	251	363	1,327	0.604	0.699	2.41	0.020	98.5
00	0.581	0.847	1.75	0.014	144	2,157	248	356	1,335	0.619	0.711	2.49	0.020	146
45	0.594	0.915	1.77	0.015	93.0	4,586	501	755	2,882	0.628	0.757	2.84	0.023	92.9
08	0.542	0.913	1.63	0.013	78.7	6,752	701	1111	3,781	0.560	0.710	3.45	0.028	78.7
44	0.440	0.687	4.29	0.035	25.0	6,383	703	1074	3,025	0.474	0.579	6.81	0.056	25.4
36	0.467	0.835	3.84	0.032	28.3	4,323	399	714	2,291	0.530	0.759	6.40	0.053	28.1
93	0.570	0.888	3.61	0.030	61.3	4,346	429	718	2,482	0.571	0.765	5.28	0.043	61.3
25	0.508	0.801	3.61	0.030	42.9	2,202	261	364	1,202	0.546	0.609	5.69	0.047	42.8
80	0.462	0.762	4.04	0.033	50.8	6,439	684	1071	3,305	0.509	0.638	7.32	0.060	50.4
34	0.511	0.794	3.53	0.029	67.0	2,197	232	363	1,182	0.538	0.673	5.13	0.042	67.4
35	0.370	0.654	4.30	0.035	31.1	13,199	1608	2229	6,710	0.508	0.563	6.32	0.052	31.5
05	0.382	0.773	9.75	0.080	18.0	9,239	663	1521	3,618	0.392	0.719	9.86	0.081	17.7
35	0.492	0.819	4.02	0.033	16.3	13,849	1318	2273	6,773	0.489	0.675	4.14	0.034	16.3
52	0.578	0.842	1.96	0.016	71.6	13,765	1456	2351	7,818	0.568	0.734	4.27	0.035	70.0
54	0.453	0.873	4.69	0.038	84.4	9,511	941	1604	5,197	0.546	0.745	5.06	0.042	81.0
73	0.460	0.894	5.54	0.045	64.5	9,910	842	1460	5,248	0.530	0.735	5.93	0.049	61.7
03	0.565	0.832	3.77	0.031	57.6	12,375	1447	2169	6,874	0.555	0.666	4.88	0.040	56.4
18	0.380	0.679	4.92	0.081	35.6	13,759	1338	2235	6,352	0.468	0.625	5.61	0.092	32.8
45	0.326	0.684	6.54	0.107	16.7	27,079	2400	4471	10,438	0.385	0.574	7.47	0.123	17.4
53	0.365	0.763	5.24	0.086	29.1	13,847	1131	2140	6,035	0.436	0.660	5.83	0.096	28.4
73	0.346	0.674	5.28	0.087	13.4	25,495	2209	4209	10,931	0.429	0.654	6.61	0.108	14.4
18	0.203	0.247	21.47	0.176	3.08	15,420	1948	2456	3,531	0.229	0.239	23.13	0.190	3.09
09	0.186	0.381	14.72	0.121	3.82	10,409	1158	1719	2,818	0.271	0.321	17.91	0.147	3.75
22	0.222	0.509	9.23	0.076	5.46	5,381	638	888	1,977	0.367	0.409	11.94	0.098	5.46
11	0.400	0.840	4.53	0.037	24.9	4,867	455	804	2,270	0.466	0.659	7.27	0.060	25.6
38	0.292	0.710	5.18	0.042	15.9	10,456	849	1726	3,584	0.343	0.558	10.93	0.090	16.1
85	0.250	0.541	9.65	0.079	13.8	15,203	1389	2510	4,793	0.315	0.456	13.46	0.110	13.8
19	0.292	0.440	8.46	0.139	5.29	15,134	1328	2499	4,419	0.292	0.440	8.46	0.139	5.29
56	0.230	0.483	8.45	0.139	6.37	10,176	1002	1680	3,429	0.337	0.452	9.26	0.152	6.35
81	0.351	0.412	7.80	0.128	7.73	5,586	637	922	1,844	0.330	0.382	7.97	0.131	7.70

3

Test No.	Track Width b, cm	Track Length ℓ, cm	Penetration Resistance Gradient G, MN/m³	Load W, N	Slip S, %	Index of Track Belt Tension t_{tb}, kPa	Pressure (kPa) in Bogie No.*							
							1	2	3	4	5	6	7	1
D-71-0082-2	30.5	61.0	0.95	17,312	-0.6	4740	617	658	667	638	--	--	--	0.
83			1.00	12,480	-5.0	5238	620	619	662	631	--	--	--	0.
84			1.02	6,689	-6.7	6259	619	619	619	624	--	--	--	0,
100	61.0	61.0	1.23	17,271	-0.5	5859	577	674	663	--	--	--	--	0.
101			1.36	12,655	-2.7	5336	611	620	668	--	--	--	--	0.
102			1.26	7,523	-1.7	5746	615	616	618	--	--	--	--	0.
19	15.2	121.9	3.89	24,943	-0.4	6665	618	--	619	--	618	--	606	0.
20			3.94	15,296	-1.5	6827	624	--	622	--	622	--	627	0.
21			4.01	5,077	-0.5	6912	619	--	620	--	620	--	620	0.
22			5.75	5,336	-0.3	6903	617	--	620	--	620	--	619	0.
23			5.72	15,393	-1.2	6917	616	--	620	--	619	--	619	0
24			5.75	25,091	-0.1	6751	616	--	627	--	620	--	625	0.
52	30.5	121.9	0.97	24,984	-0.7	5917	615	--	618	--	601	--	641	0.
53			1.10	15,478	-0.4	6749	621	--	614	--	654	--	647	0.
54			0.99	5,332	-2.8	6556	618	--	620	--	655	--	624	0.
64	61.0	121.9	0.97	5,123	-1.7	7431	608	--	618	--	612	--	618	0.
65			1.03	14,997	-1.4	7467	616	--	619	--	616	--	640	0.
66			1.04	25,173	-1.0	6509	624	--	620	--	616	--	652	0.

* Road bogies are numbered consecutively, starting at the rear end of the track.

** Here, trim angle θ is defined as $\theta = \sin^{-1}(z_F/1\ell)$ where θ is considered to have a negative valu

† $\left(\frac{d}{\ell/2}\right)^n$ is not included as part of the sand-track mobility number (equation 4) for tests in this tab

Table 9

Towed Tests of Single Tracks in Mortar Sand, First Pass

in Bogie No.*			Deflection (c) in cm of Bogie No.							Towed Force P_T, N	Sinkage at Front Bogie z_F, cm	Trim** Angle θ, deg	Basic-Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$	Sand-Trac Mobility Number $\frac{G(b\ell)^{3/2}}{W} \cdot ($
5	6	7	1	2	3	4	5	6	7					
--	--	--	0.0	2.1	5.7	7.3	0.0	0.0	0.0	3466	3.4	-3.2	4.4	4.1
--	--	--	0.0	0.0	1.3	2.3	0.0	0.0	0.0	2994	5.1	-4.8	6.4	5.0
--	--	--	0.0	0.0	0.0	0.2	0.0	0.0	0.0	2227	4.6	-4.3	12.2	7.0
--	--	--	0.2	2.3	5.9	0.0	0.0	0.0	0.0	3067	3.8	-3.6	16.2	15.0
--	--	--	0.0	0.1	2.5	0.0	0.0	0.0	0.0	2697	4.3	-4.0	24.4	19.4
--	--	--	0.0	0.0	0.1	0.0	0.0	0.0	0.0	1520	3.7	-3.5	38.0	23.2
618	--	606	0.0	0.0	0.1	0.0	0.0	0.0	2.4	7104	11.8	-5.6	12.2	10.3
622	--	627	0.0	0.0	0.0	0.0	0.0	0.0	0.1	2466	6.0	-2.8	20.1	13.2
620	--	620	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1054	3.4	-1.6	61.8	23.5
620	--	619	0.0	0.0	0.0	0.0	0.0	0.0	0.0	951	3.2	-1.5	84.3	32.9
619	--	619	0.0	0.0	0.0	0.0	0.0	0.0	0.1	1998	4.0	-1.9	29.1	19.2
620	--	625	0.0	0.0	0.1	0.0	0.0	0.0	0.3	3205	5.0	-2.4	17.9	15.1
601	--	641	0.0	0.0	0.0	0.0	0.0	0.0	1.6	3685	6.0	-2.8	8.8	7.4
654	--	647	0.0	0.0	0.0	0.0	0.0	0.0	0.9	2587	5.6	-2.6	17.7	11.7
655	--	624	0.0	0.0	0.0	0.0	0.0	0.0	0.1	1071	5.2	-2.5	42.1	16.4
612	--	618	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1012	3.0	-1.4	121.4	46.2
616	--	640	0.0	0.0	0.0	0.0	0.0	0.0	0.6	2421	3.1	-1.4	44.0	28.7
616	--	652	0.0	0.0	0.0	0.0	0.0	0.0	1.4	3029	2.5	-1.2	26.5	22.4

have a negative value when the front end of the track is lower than the rear end.

or tests in this table because for each test $\frac{d}{\ell/2}$ had a value of 1.0.

2

, First Pass

e No.		Towed Force P_T, N	Sinkage at Front Bogie z_F, cm	Trim** Angle θ, deg	Basic-Variable Prediction Term $\frac{G(b\ell)^{3/2}}{W}$	Sand-Track Mobility Number† $\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$	Towed Force Coefficient $\frac{P_T}{W}$	Sinkage Coefficient $\frac{z_F}{\ell}$
5	7							
0.0	0.0	3466	3.4	-3.2	4.4	4.1	0.200	0.056
0.0	0.0	2994	5.1	-4.8	6.4	5.0	0.240	0.084
0.0	0.0	2227	4.6	-4.3	12.2	7.0	0.333	0.075
0.0	0.0	3067	3.8	-3.6	16.2	15.0	0.178	0.062
0.0	0.0	2697	4.3	-4.0	24.4	19.4	0.213	0.070
0.0	0.0	1520	3.7	-3.5	38.0	23.2	0.202	0.061
0.0	2.4	7104	11.8	-5.6	12.2	10.3	0.285	0.097
0.0	0.1	2466	6.0	-2.8	20.1	13.2	0.161	0.049
0.0	0.0	1054	3.4	-1.6	61.8	23.5	0.207	0.028
0.0	0.0	951	3.2	-1.5	84.3	32.9	0.178	0.026
0.0	0.1	1998	4.0	-1.9	29.1	19.2	0.130	0.033
0.0	0.3	3205	5.0	-2.4	17.9	15.1	0.128	0.041
0.0	1.6	3685	6.0	-2.8	8.8	7.4	0.147	0.049
0.0	0.9	2587	5.6	-2.6	17.7	11.7	0.167	0.046
0.0	0.1	1071	5.2	-2.5	42.1	16.4	0.201	0.043
0.0	0.0	1012	3.0	-1.4	121.4	46.2	0.198	0.025
0.0	0.6	2421	3.1	-1.4	44.0	28.7	0.161	0.025
0.0	1.4	3029	2.5	-1.2	26.5	22.4	0.120	0.021

track is lower than the rear end.

d / 1/2 had a value of 1.0.

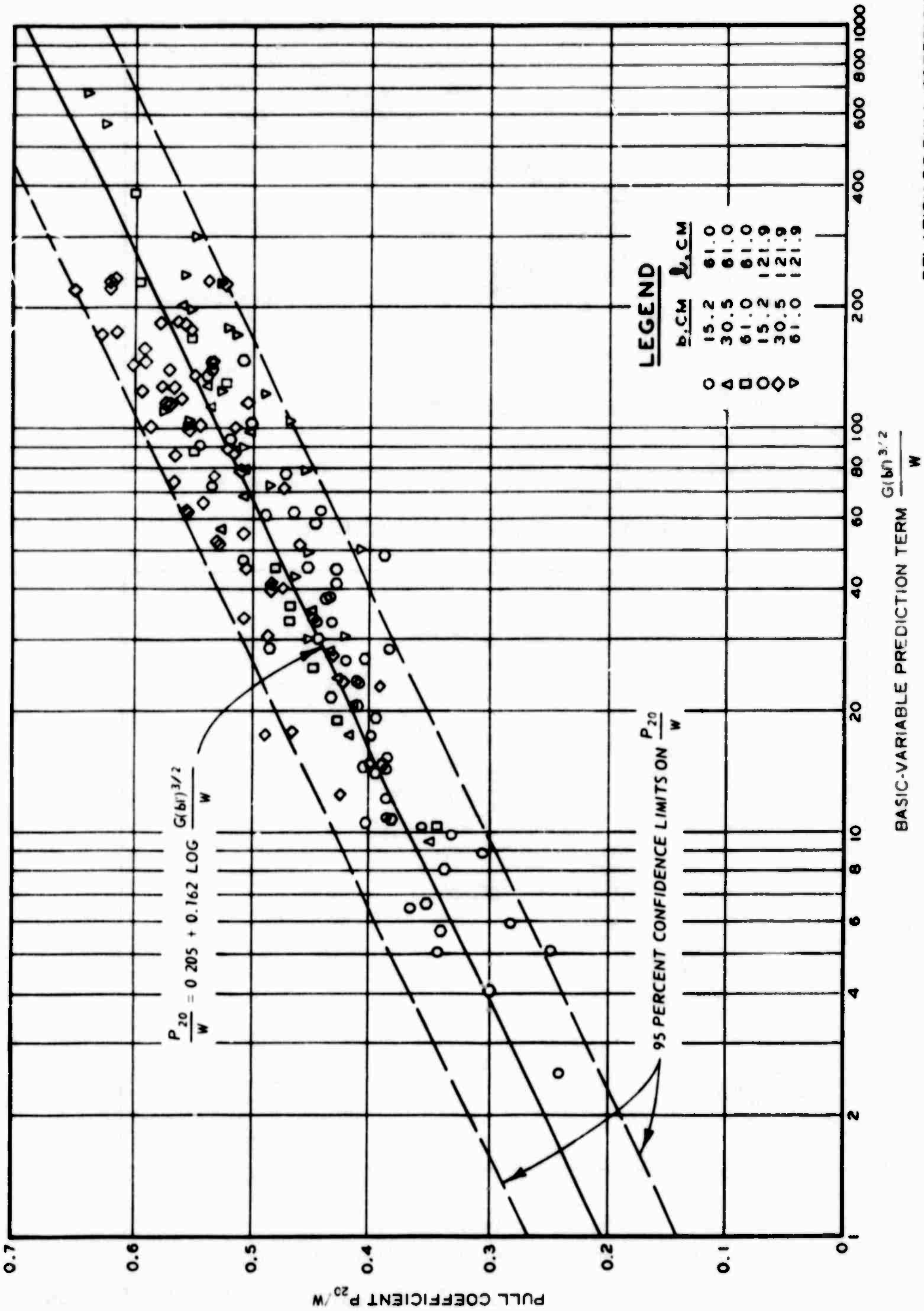
Table 10

Prototype Tracked-Vehicle Tests in Mortar Sand, 20 Percent Slip, First Pass

Test Numbers	Vehicle	Before-Traffic Penetration Resistance Gradient G, MN/m^3	Track Width b, cm	Track Length ℓ, cm	Load* W, N	Pull at 20 Percent Slip P_{20}, N: First Test	Second Test	Avg	Average Pull Coefficient P_{20}/W	$\frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{max}}\right)^{1/2}$**
F-73-0004, 5-2	CD4	0.76	40.6	182.9	44,670	18,280	20,820	19,550	0.438	10.9
F-73-0006, 7-2	CD4	1.98	40.6	182.9	44,670	20,370	20,780	20,575	0.461	28.4
F-73-0008, 9-2	CD4	3.50	40.6	182.9	44,670	20,680	21,180	20,930	0.469	50.1
F-73-0010, 11-2	CD4	5.00	40.6	182.9	44,670	21,000	21,400	21,200	0.475	71.6
F-73-0012, 13-2	M29C	0.80	50.8	198.1	13,255	7,490	7,260	7,375	0.556	38.5
F-73-0014, 15-2	M29C	2.10	50.8	198.1	13,255	7,500	7,450	7,475	0.564	101
F-73-0016, 17-2	M113A1	0.75	38.1	266.7	43,725	17,360	17,100	17,230	0.394	11.1
F-73-0018, 19-2	M113A1	2.10	38.1	266.7	43,725	22,120	19,680	20,900	0.478	31.1
F-73-0028, 29-2	M48A1	0.80	71.1	401.3	212,235	77,040	68,560	72,800	0.343	11.5

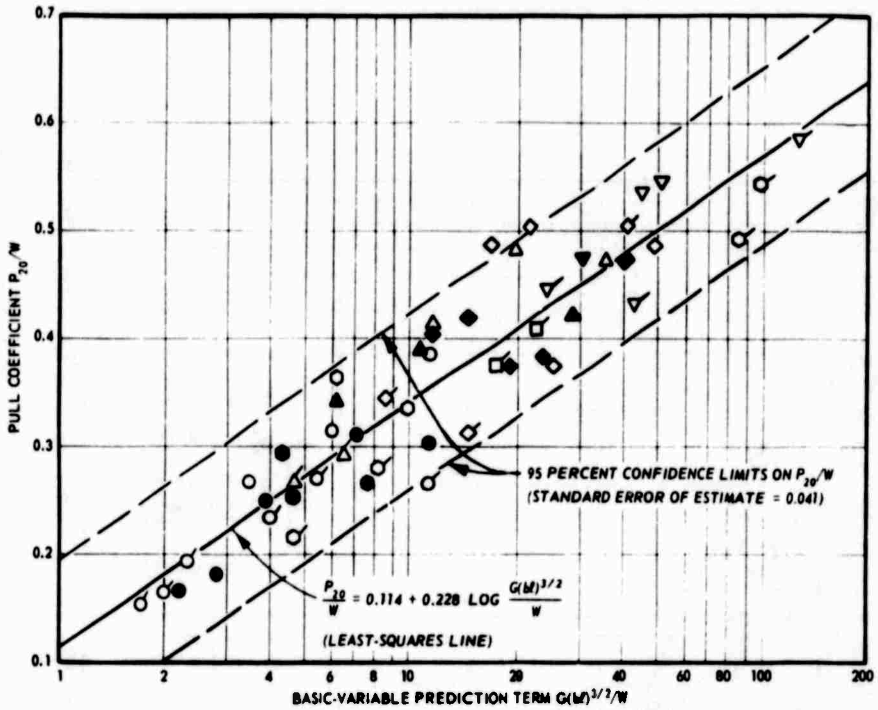
* "W" equals one-half of total vehicle weight.

** W/W_{max} equals 1.0 for the CD4, and 0.4 for the M29C, M113A1, and M48A1.

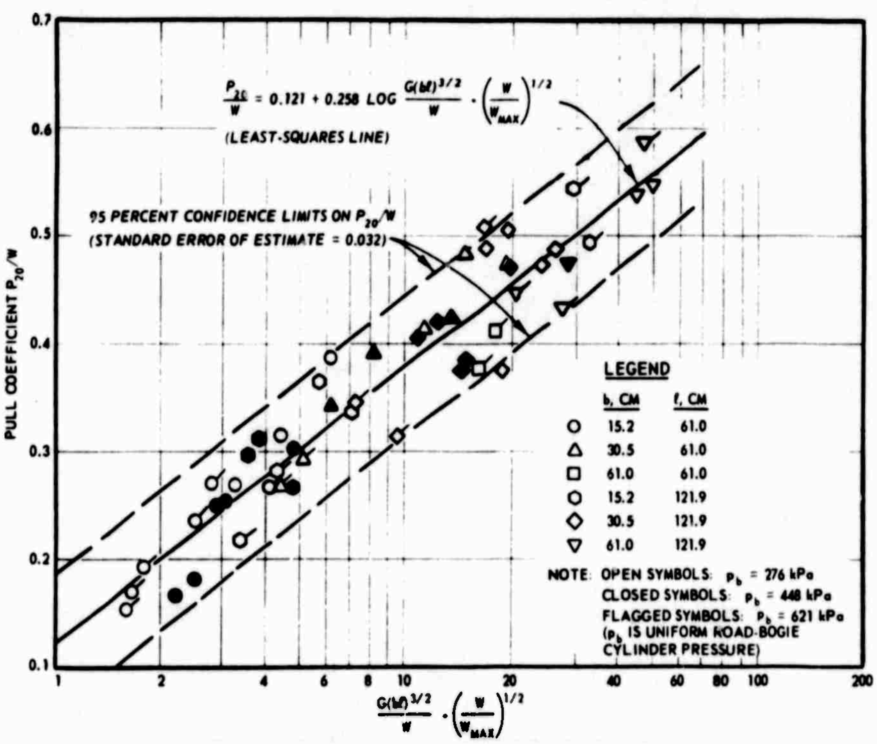


RELATION OF PULL COEFFICIENT TO LOG (BASIC-VARIABLE PREDICTION TERM)

20-PERCENT SLIP POINT
MODEL TRACK TESTS
AIR-DRY YUMA SAND
(FROM REFERENCE 2)



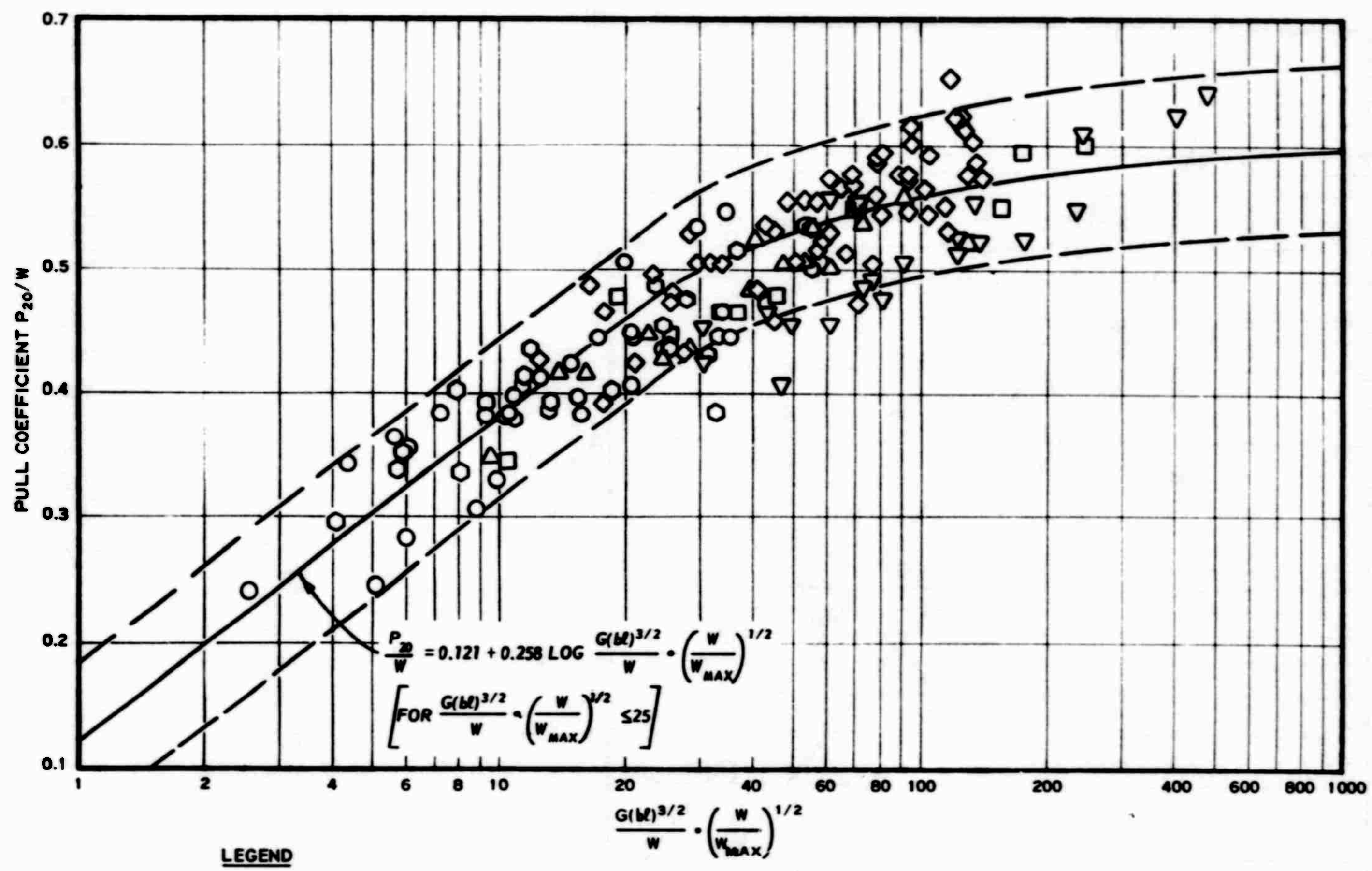
a. NO CORRECTION FOR SUSPENSION CHARACTERISTICS



b. PREDICTION TERM CORRECTED FOR SUSPENSION IN TERMS OF $(W/W_{MAX})^{1/2}$

USE OF $(W/W_{MAX})^{1/2}$ TO CHARACTERIZE UNIFORM TRACK SUSPENSIONS

20-PERCENT SLIP POINT
THREE VALUES OF ROAD-BOGIE CYLINDER PRESSURE
MODEL TRACK TESTS
AIR-DRY MORTAR SAND



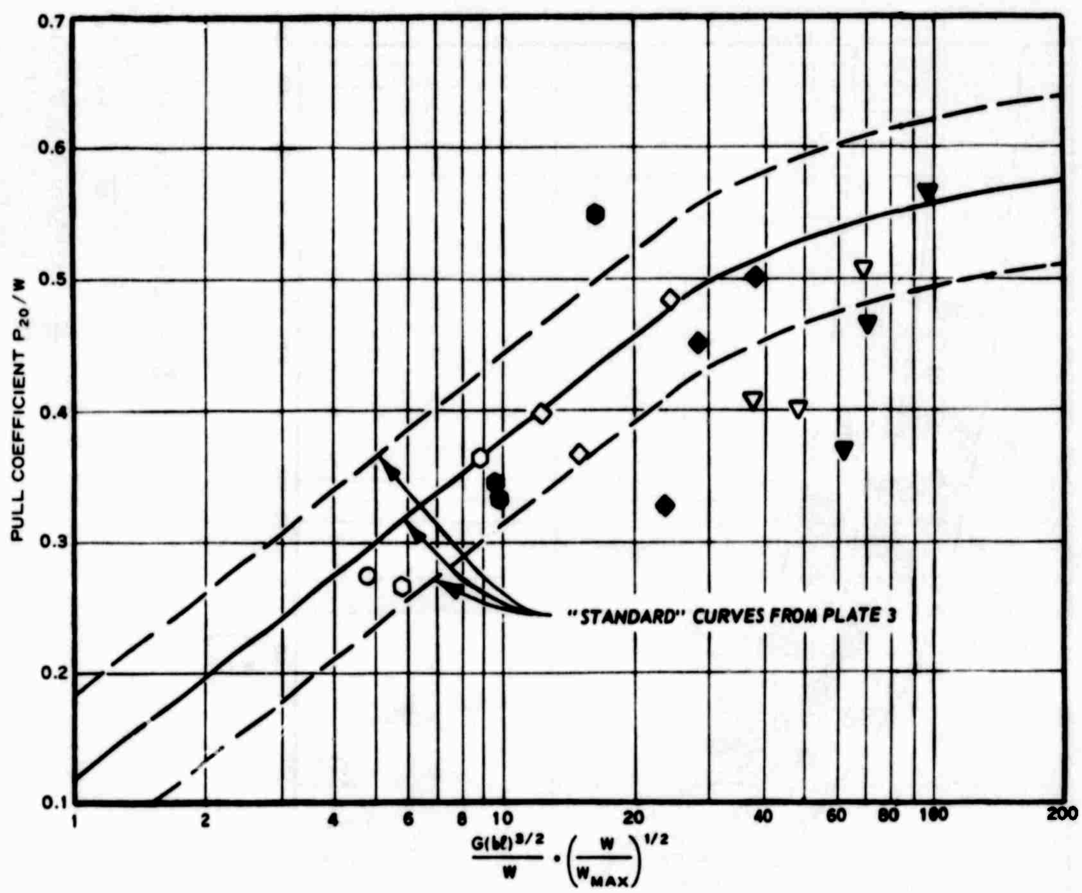
LEGEND

	b, CM	ℓ, CM
○	15.2	61.0
△	30.5	61.0
□	61.0	61.0
⬡	15.2	121.9
◇	30.5	121.9
▽	61.0	121.9

RELATION OF PULL COEFFICIENT TO

$\text{LOG } \frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{MAX}}\right)^{1/2}$

20-PERCENT SLIP POINT
MODEL TRACK TESTS
AIR-DRY YUMA SAND



LEGEND

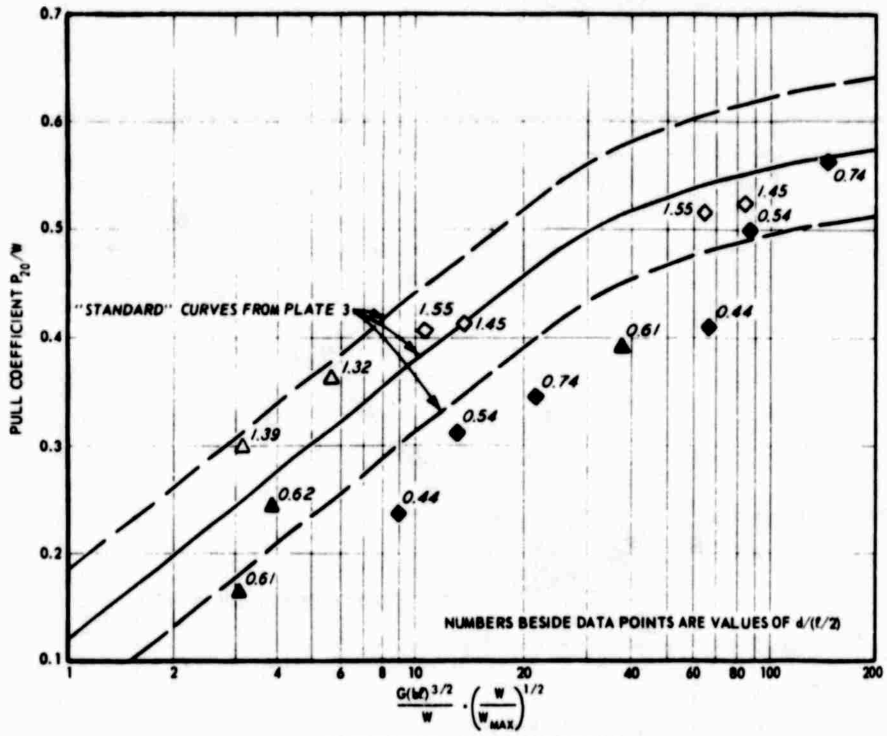
	b, CM	ℓ, CM
○	15.2	121.9
◇	30.5	121.9
▽	61.0	121.9

NOTE: OPEN SYMBOLS: s_w/d_w = 40.6 CM/17.8 CM = 2.3
CLOSED SYMBOLS: s_w/d_w = 61.0 CM/17.8 CM = 3.4

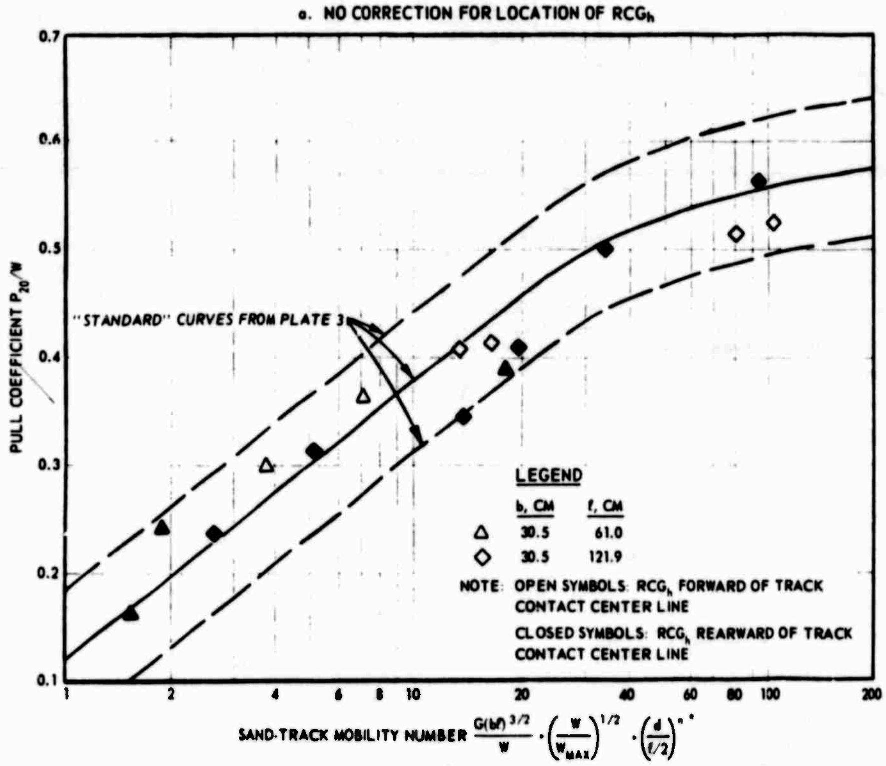
INFLUENCE OF ROAD-WHEEL SPACING ON THE RELATION OF PULL COEFFICIENT TO

$$\text{LOG } \frac{G(b\ell)^{3/2}}{W} \cdot \left(\frac{W}{W_{MAX}}\right)^{1/2}$$

20-PERCENT SLIP POINT
ROAD-WHEEL SPACINGS OF 40.6 AND 61.0 CM
MODEL TRACK TESTS
AIR-DRY YUMA SAND



a. NO CORRECTION FOR LOCATION OF RCG_h

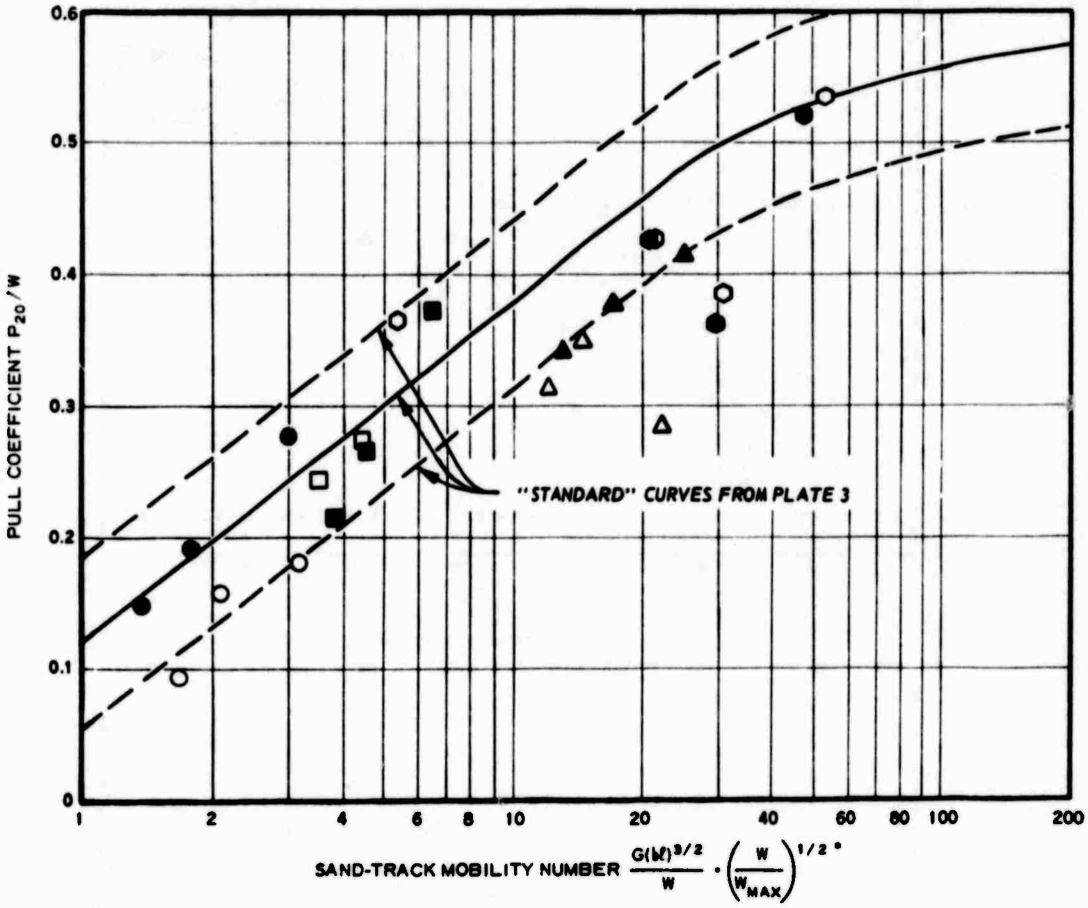


b. MOBILITY NUMBER CORRECTED FOR LOCATION OF RCG_h

* $n = 3/2$ FOR RCG_h REARWARD OF CENTER LINE
$n = 1/2$ FOR RCG_h FORWARD OF CENTER LINE

INFLUENCE OF RCG_h LOCATION ON THE RELATION OF P_{20}/W VERSUS LOG (SAND-TRACK MOBILITY NUMBER)

20-PERCENT SLIP POINT
RCG_h LOCATIONS FORWARD AND REARWARD OF CENTER LINE
MODEL TRACK TESTS
AIR-DRY YUMA SAND



* EACH DATA POINT IN THIS PLATE HAS A VALUE OF $d/(\ell/2)$ = 1.0 IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4)

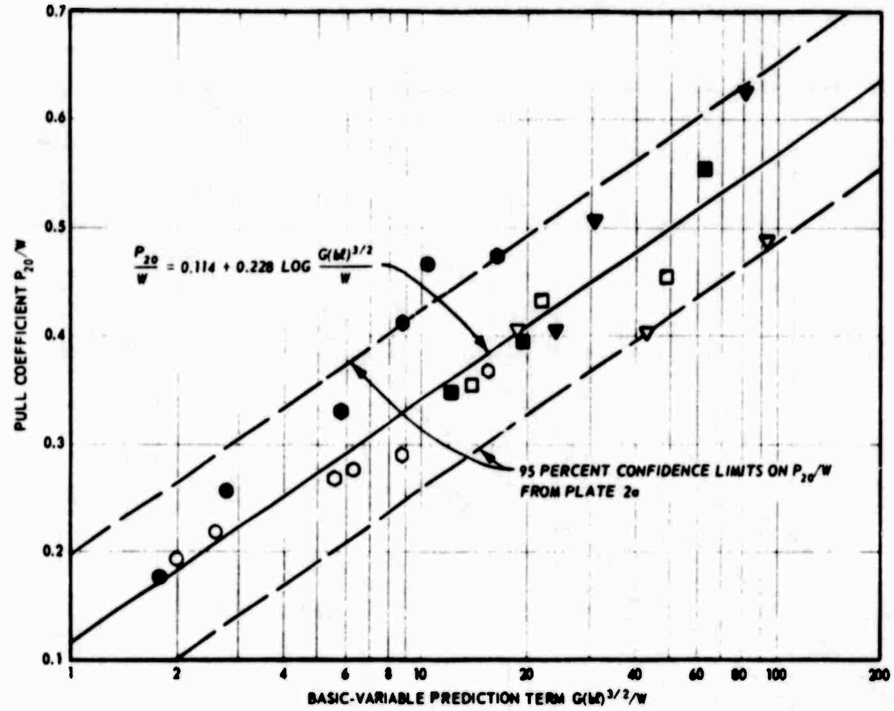
LEGEND

	b, CM	ℓ, CM
○	15.2	61.0
△	61.0	61.0
□	15.2	121.9
○	61.0	121.9

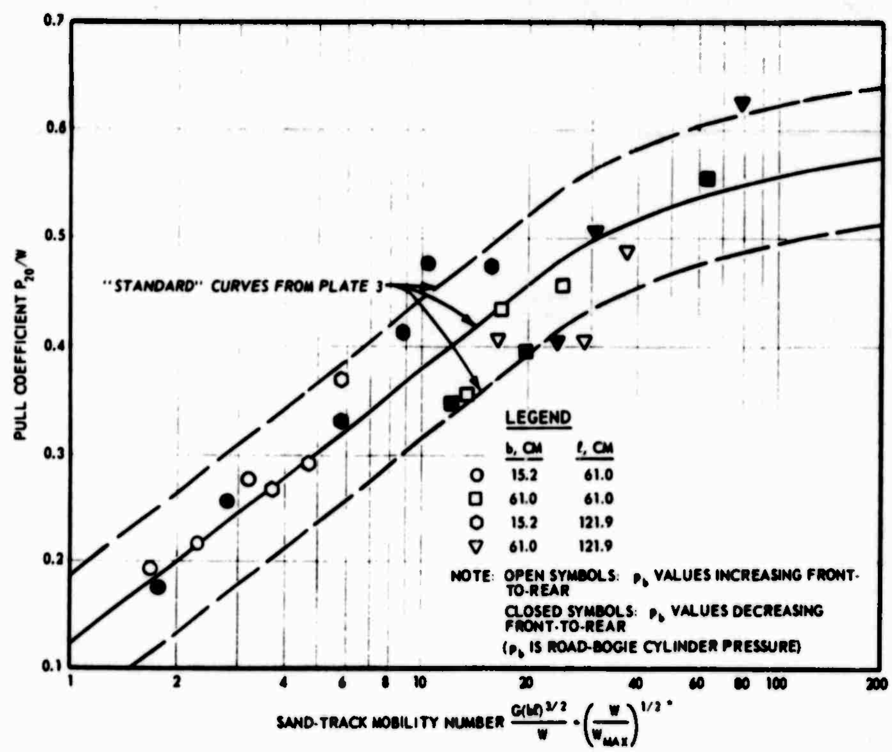
NOTE: OPEN SYMBOLS: INDEX OF TRACK-BELT TENSION = 1380 kPa
CLOSED SYMBOLS: INDEX OF TRACK-BELT TENSION = 4140 kPa

INFLUENCE OF TRACK-BELT TENSION ON THE RELATION OF PULL COEFFICIENT TO LOG (SAND-TRACK MOBILITY NUMBER)

20-PERCENT SLIP POINT
MODEL TRACK TESTS
AIR-DRY YUMA SAND



a. NO CORRECTION FOR DISTRIBUTION OF PRESSURE IN ROAD-BOGIE CYLINDERS

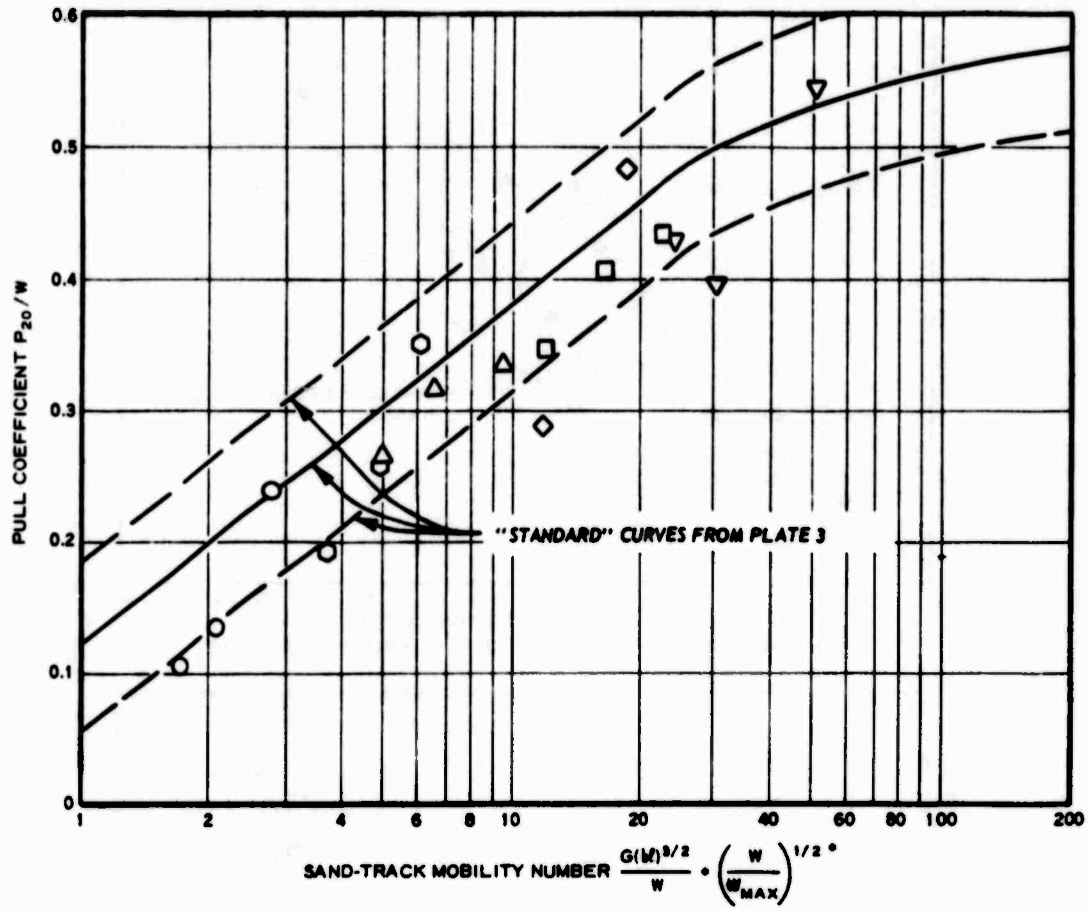


b. PREDICTION TERM CORRECTED FOR DISTRIBUTION OF PRESSURE IN ROAD-BOGIE CYLINDERS

* VALUES OF W_{MAX} ARE COMPUTED ON THE BASIS OF PRESSURE IN THE REARMOST ROAD-BOGIE CYLINDER ONLY. ALSO, EACH DATA POINT IN THIS PLATE HAS A VALUE OF $d/(\ell/2) = 1.0$ IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4).

USE OF $(W/W_{MAX})^{1/2}$ TO CHARACTERIZE LINEARLY DISTRIBUTED TRACK SUSPENSIONS

20-PERCENT SLIP POINT
LINEARLY-INCREASING AND LINEARLY-DECREASING PATTERNS OF ROAD-BOGIE CYLINDER PRESSURE
MODEL TRACK TESTS
AIR-DRY YUMA SAND



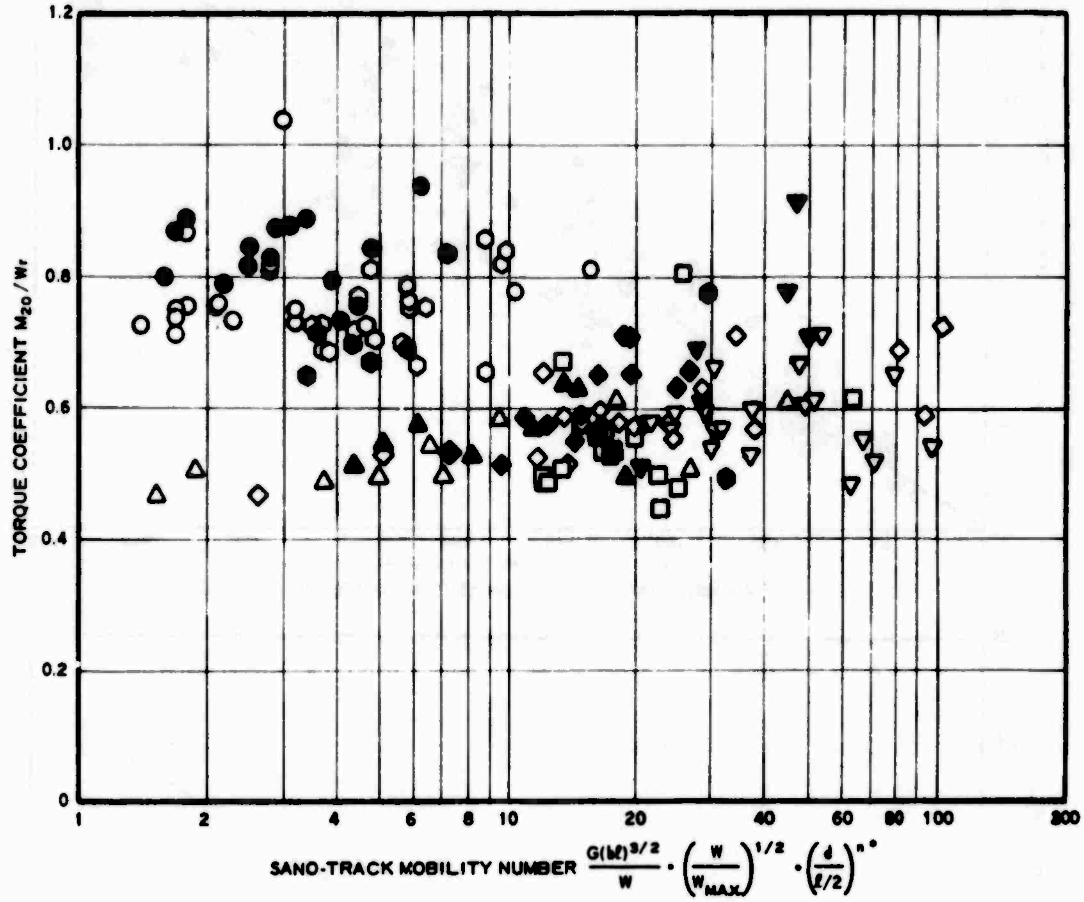
* EACH DATA POINT IN THIS PLATE HAS A VALUE OF $d/(\ell/2)=1.0$ IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4).

LEGEND

	b, CM	ℓ, CM
○	15.2	61.0
△	30.5	61.0
□	61.0	61.0
○	15.2	121.9
◇	30.5	121.9
▽	61.0	121.9

INFLUENCE OF FRONT-SPROCKET DRIVE ON THE RELATION OF PULL COEFFICIENT TO LOG (SAND-TRACK MOBILITY NUMBER)

20-PERCENT SLIP POINT
MODEL TRACK TESTS
AIR-DRY YUMA SAND



* EXCEPT FOR TABLE 4, ALL TESTS IN TABLES 2 THROUGH 7 HAVE $d/(\ell/2) = 1.0$ (i.e. RCG_h AT TRACK GEOMETRIC CENTER LINE). FOR TESTS IN TABLE 4, $n = 3/2$ IN THE SAND-TRACK MOBILITY NUMBER FOR $d/(\ell/2) < 1.0$ (i.e. RCG_h REARWARD OF CENTER LINE), AND $n = 1/2$ FOR $d/(\ell/2) > 1.0$ (RCG_h FORWARD OF CENTER LINE).

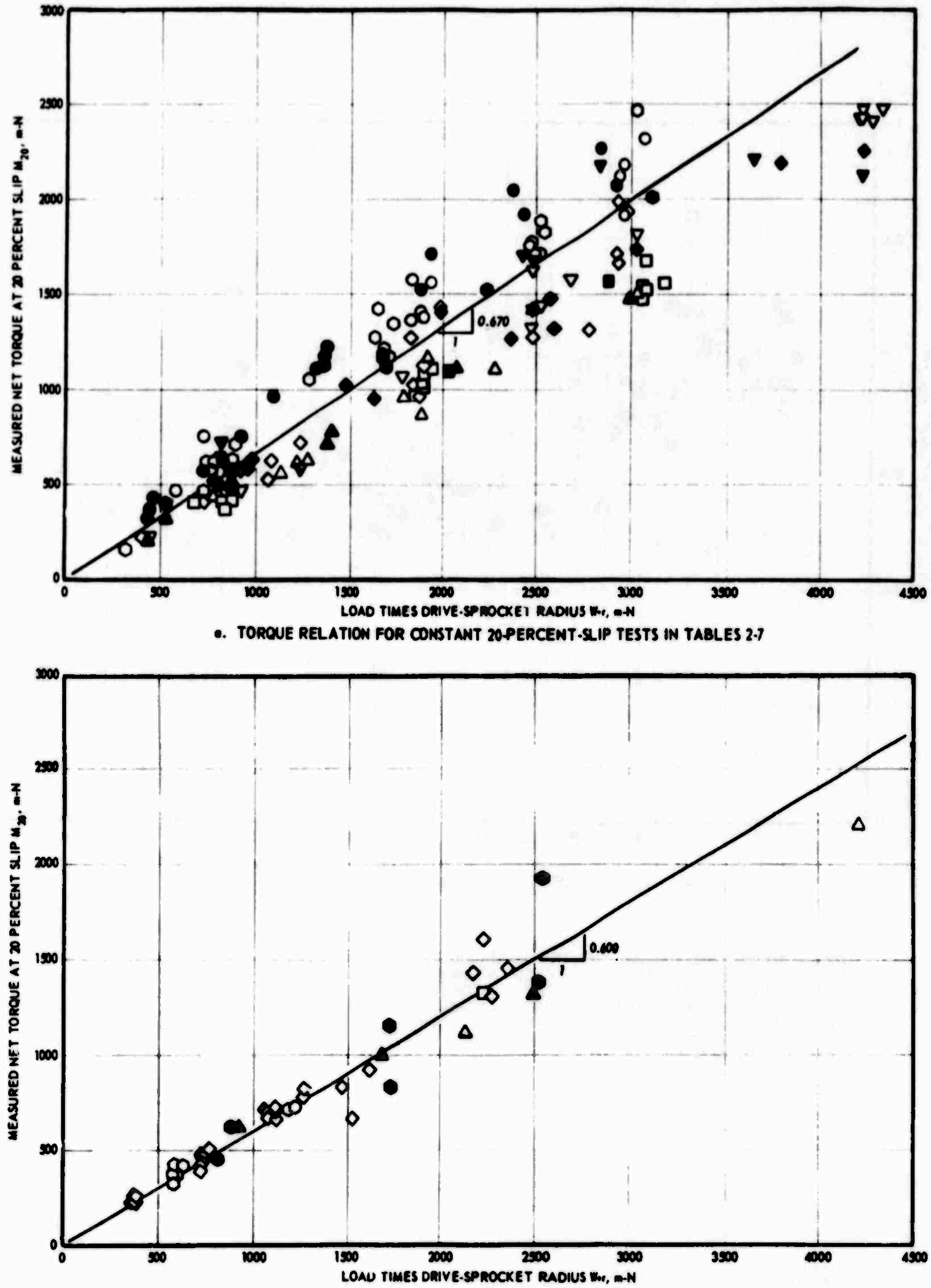
LEGEND

	d, CM	ℓ, CM
○	15.2	61.0
△	30.5	61.0
□	61.0	61.0
○	15.2	121.9
◇	30.5	121.9
▽	61.0	121.9

NOTE: OPEN SYMBOLS: YUMA SAND DATA
CLOSED SYMBOLS: MORTAR SAND DATA

RELATION OF TORQUE COEFFICIENT TO LOG (SAND-TRACK MOBILITY NUMBER)

20-PERCENT SLIP POINT
MODEL TRACK TESTS
AIR-DRY YUMA AND MORTAR SANDS



a. TORQUE RELATION FOR CONSTANT 20-PERCENT-SLIP TESTS IN TABLES 2-7

b. TORQUE RELATION FOR 20-PERCENT-SLIP POINT OF PROGRAMMED-INCREASING-SLIP TESTS (TABLE 8)

LEGEND

	b, CM	r, CM
○	15.2	61.0
△	30.5	61.0
□	61.0	61.0
○	15.2	121.9
◇	30.5	121.9
▽	61.0	121.9

NOTE: OPEN SYMBOLS: YUMA SAND DATA
CLOSED SYMBOLS: MORTAR SAND DATA

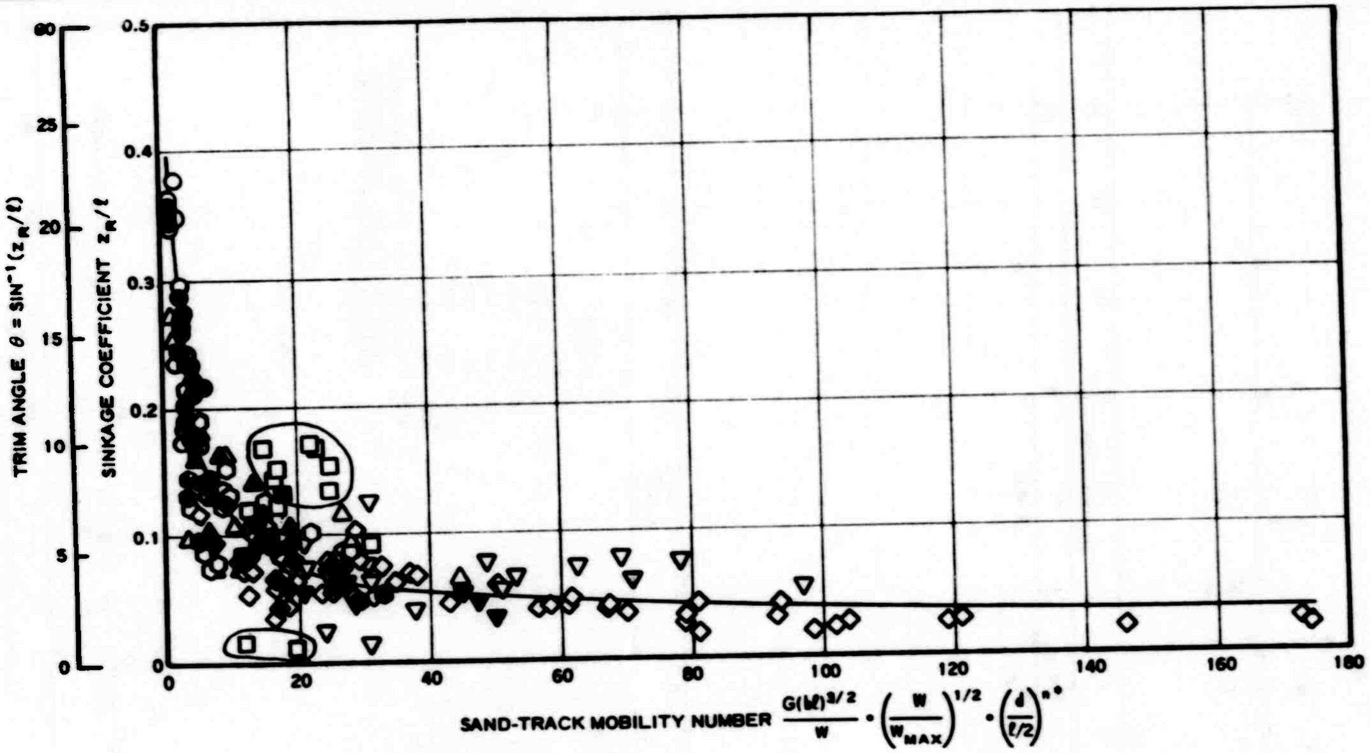
RELATION OF MEASURED NET TORQUE TO LOAD TIMES DRIVE-SPROCKET RADIUS

20-PERCENT SLIP POINT

CONSTANT-20-PERCENT-SLIP AND PROGRAMMED-INCREASING-SLIP TESTS

MODEL TRACK TESTS

AIR-DRY YUMA AND MORTAR SANDS



* $d/(\ell/2) = 1.0$ FOR ALL TESTS EXCEPT THOSE IN TABLE 4. IN TABLE 4, $n = 3/2$ FOR $d/(\ell/2) < 1.0$ (RCG_h REARWARD OF TRACK GEOMETRIC CENTER LINE), AND $n = 1/2$ FOR $d/(\ell/2) > 1.0$ (RCG_h FORWARD OF CENTER LINE).

LEGEND

	b, CM	ℓ, CM
○	15.2	61.0
△	30.5	61.0
□	61.0	61.0
⬡	15.2	121.9
◇	30.5	121.9
▽	61.0	121.9

NOTE: OPEN SYMBOLS = YUMA SAND DATA
CLOSED SYMBOLS = MORTAR SAND DATA

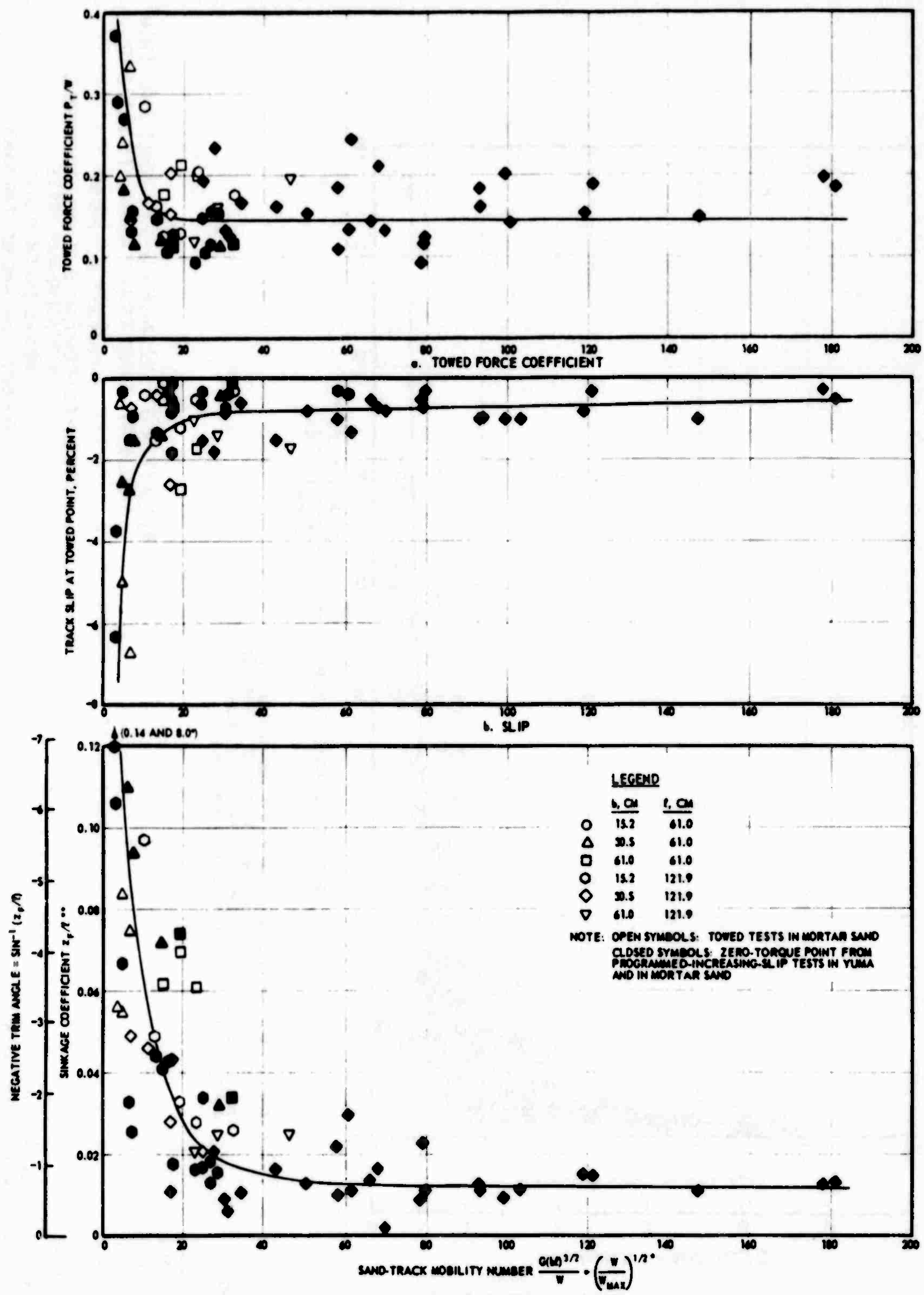
RELATION OF SINKAGE COEFFICIENT AND TRIM ANGLE TO SAND-TRACK MOBILITY NUMBER

20-PERCENT SLIP POINT

CONSTANT-20-PERCENT-SLIP AND PROGRAMMED-INCREASING-SLIP TESTS

MODEL TRACK TESTS

AIR-DRY YUMA AND MORTAR SANDS

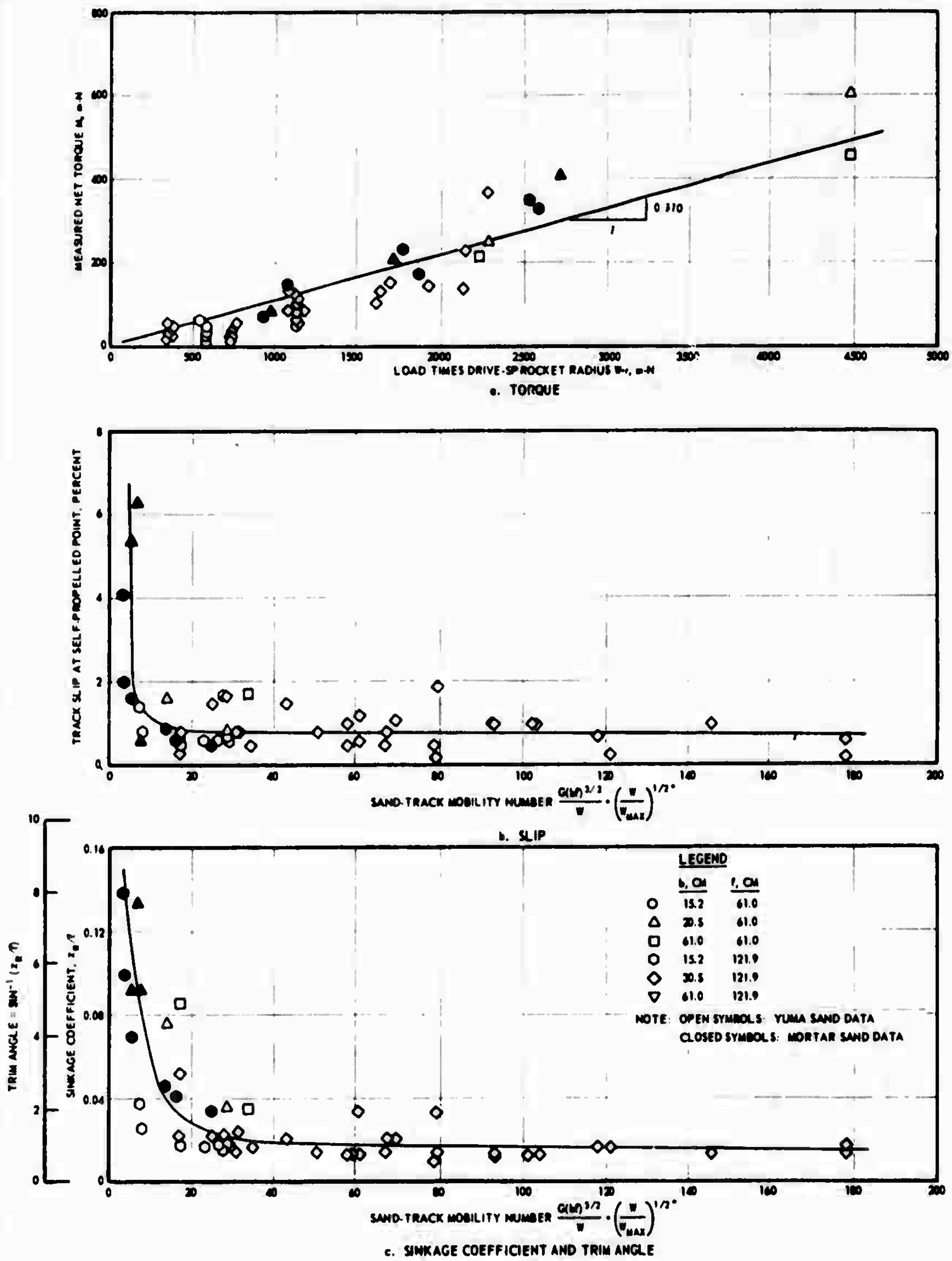


* EACH DATA POINT IN THIS PLATE HAS A VALUE OF $d/(\ell/2) = 1.0$ IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4).

** z_p = TRACK SINKAGE AT FRONT ROAD WHEEL

RELATIONS OF TOWED FORCE COEFFICIENT, SLIP, SINKAGE COEFFICIENT, AND TRIM ANGLE TO SAND-TRACK MOBILITY NUMBER

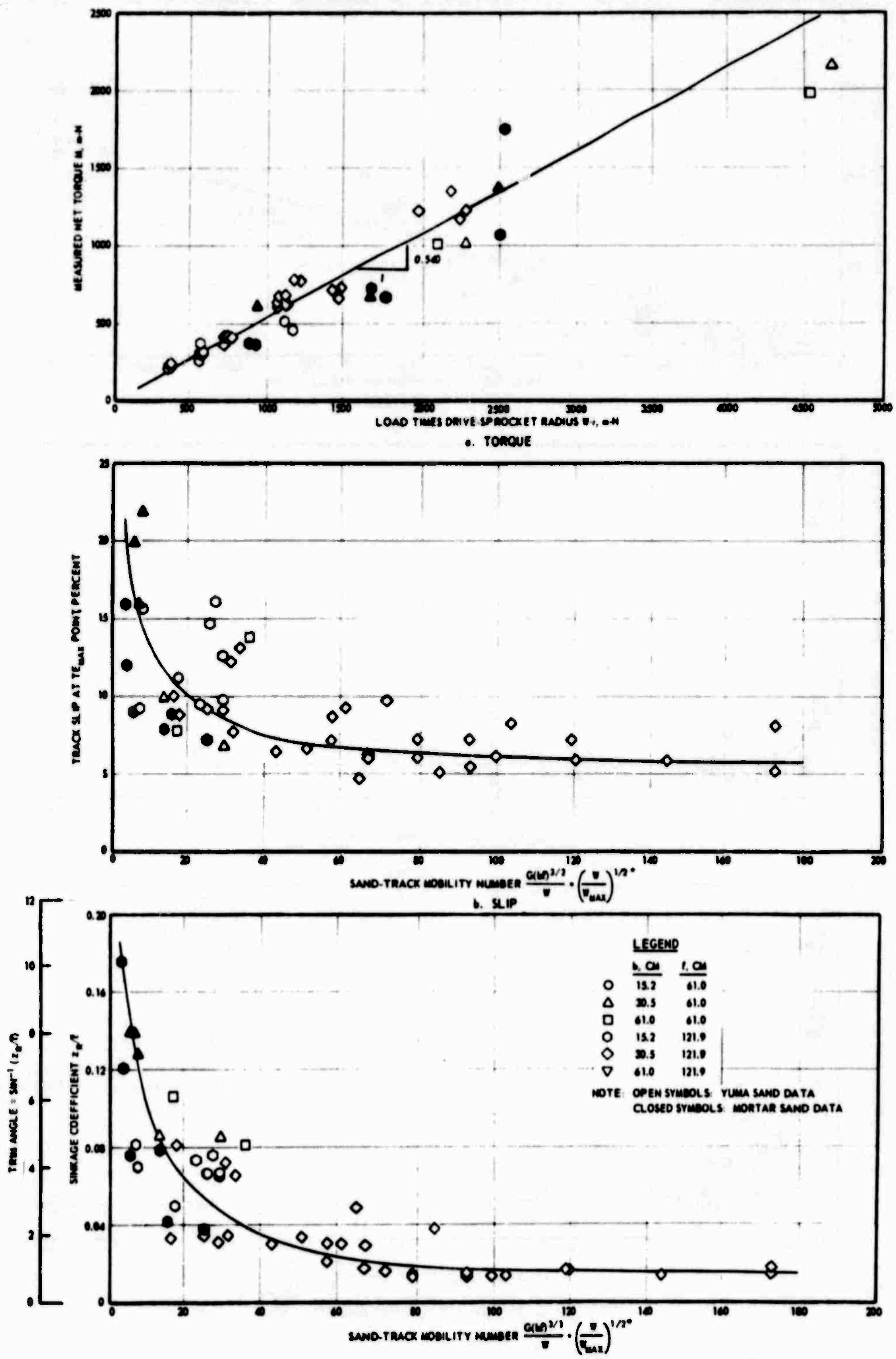
TOWED CONDITION
MODEL TRACK TESTS
TOWED AND PROGRAMMED-INCREASING-SLIP TESTS
AIR-DRY YUMA AND MORTAR SANDS



* EACH DATA POINT IN THIS PLATE HAS A VALUE OF $d/(\ell/2) = 1.0$ IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4).

TORQUE, SLIP, SINKAGE, AND TRIM ANGLE RELATIONS AT THE SELF-PROPELLED POINT

MODEL TRACK TESTS
PROGRAMMED-INCREASING-SLIP TESTS
AIR-DRY YUMA AND MORTAR SANDS

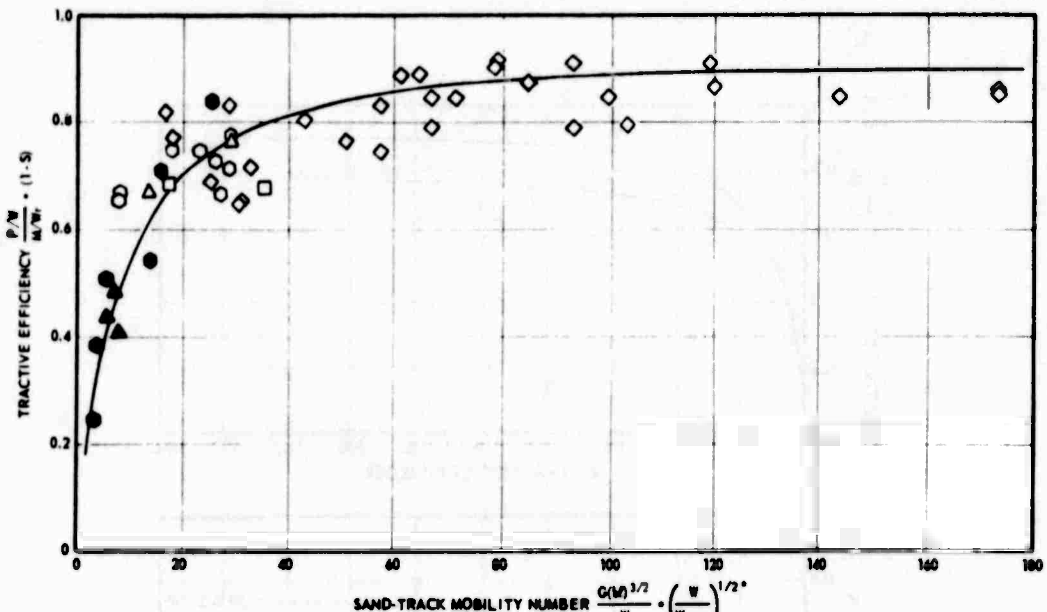


c. SINKAGE COEFFICIENT AND TRIM ANGLE

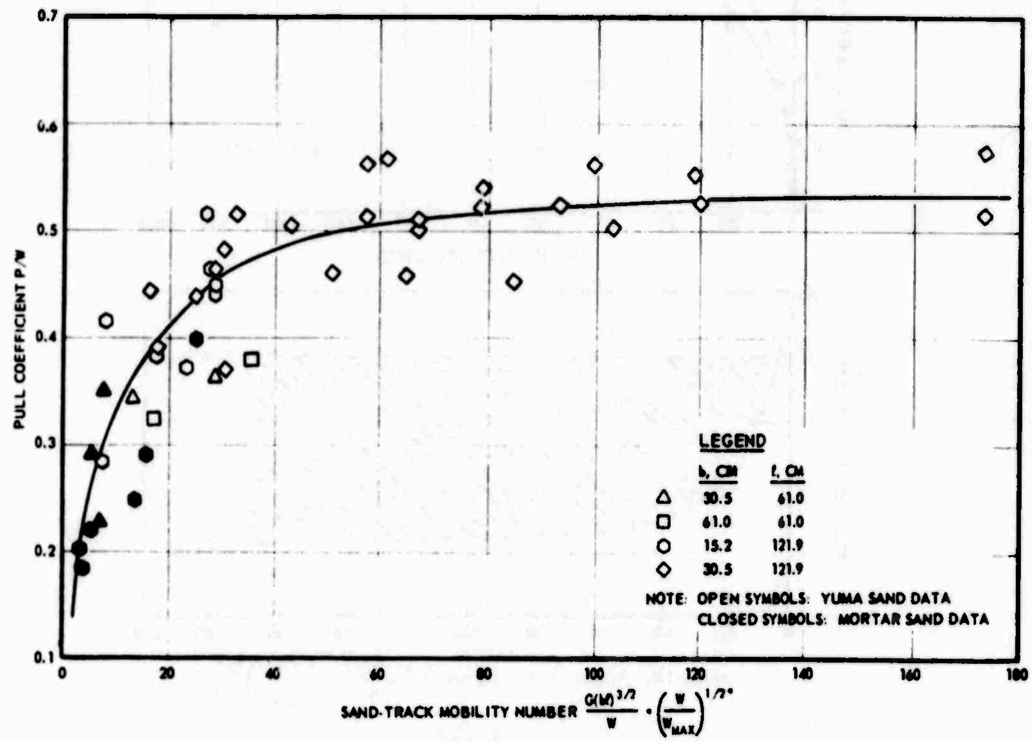
* EACH DATA POINT IN THIS PLATE HAS A VALUE OF $\delta/(l/2) = 1.0$ IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4).

TORQUE, SLIP, SINKAGE, AND TRIM ANGLE RELATIONS AT THE MAXIMUM-TRACTIVE-EFFICIENCY POINT

MODEL TRACK TESTS
PROGRAMMED-INCREASING-SLIP TESTS
AIR-DRY YUMA AND MORTAR SANDS



a. TRACTIVE EFFICIENCY

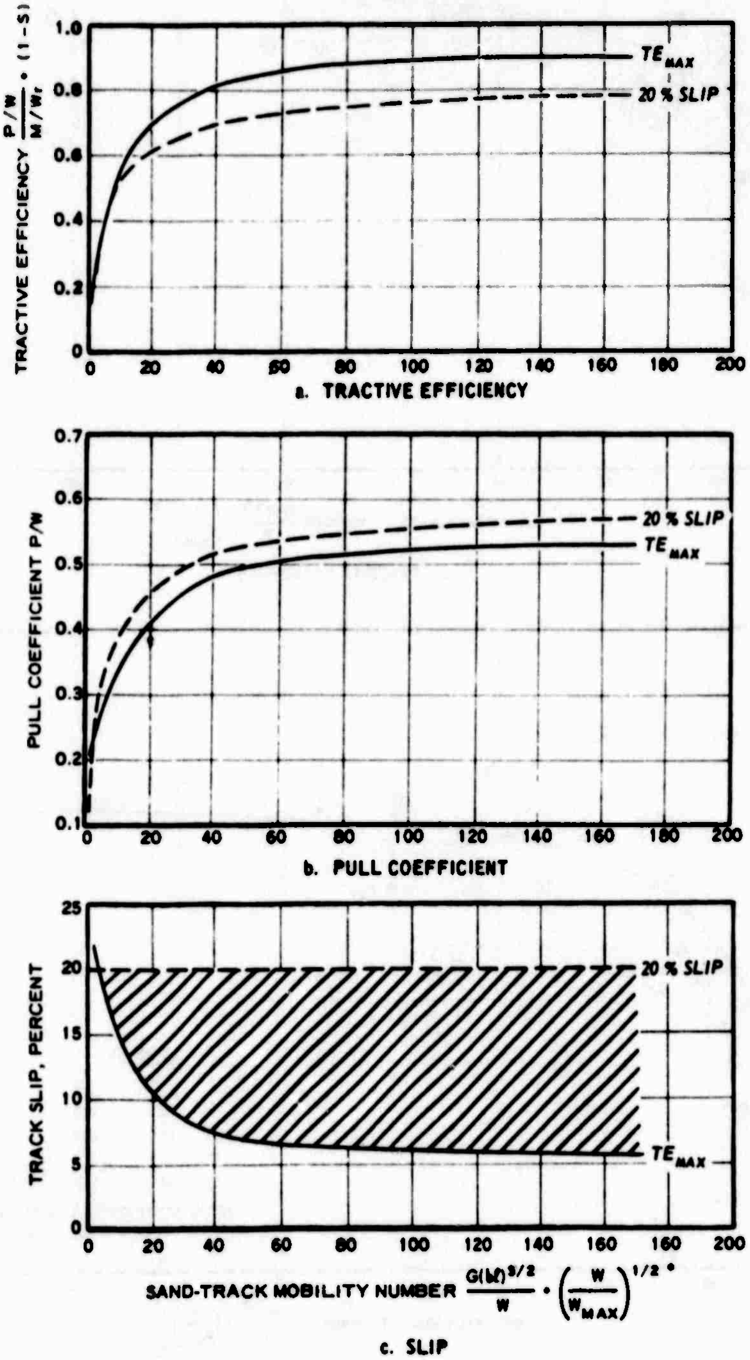


b. PULL COEFFICIENT

* EACH DATA POINT IN THIS PLATE HAS A VALUE OF $d/(\ell/2) = 1.0$ IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4).

RELATIONS OF TRACTIVE EFFICIENCY AND PULL COEFFICIENT TO SAND-TRACK MOBILITY NUMBER AT THE TE_{MAX} POINT

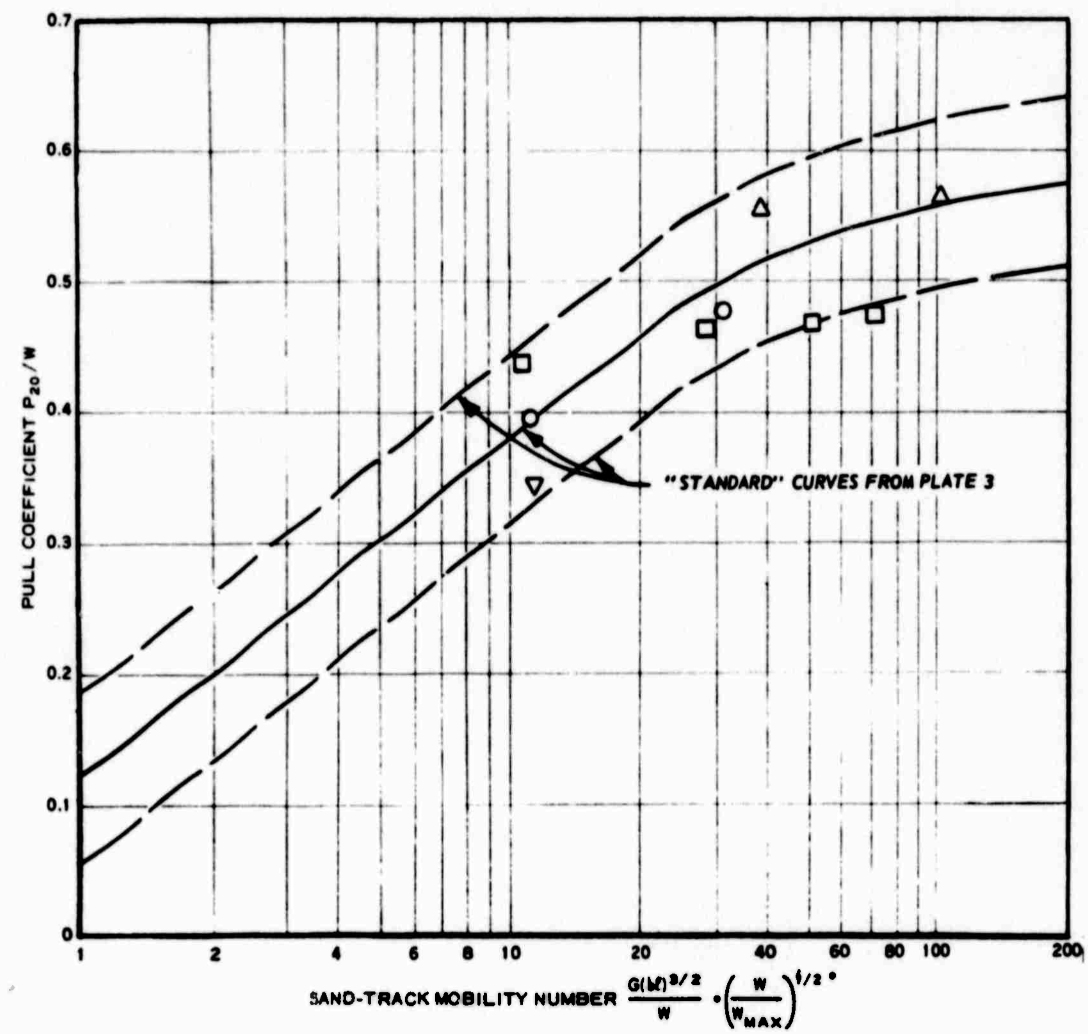
MODEL TRACK TESTS
PROGRAMMED-INCREASING-SLIP TESTS
AIR-DRY YUMA AND MORTAR SANDS



* THE RELATIONS IN THIS PLATE WERE DEVELOPED WITH DATA WHOSE VALUE OF $\delta/(\ell/2)$ = 1.0 IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4).

RELATIONS OF TRACTIVE EFFICIENCY, PULL COEFFICIENT, AND SLIP TO SAND-TRACK MOBILITY NUMBER

MODEL TRACK TESTS
20-PERCENT-SLIP AND TE_{MAX} POINTS
AIR-DRY YUMA AND MORTAR SANDS



* THE AT-REST HORIZONTAL CENTER OF GRAVITY WAS VERY NEAR THE TRACK GEOMETRIC CENTER LINE FOR EACH TEST VEHICLE, SO THAT $d/(l/2)$ COULD BE TAKEN EQUAL TO 1.0 IN THE SAND-TRACK MOBILITY NUMBER (EQUATION 4).

LEGEND

- ○ M113A1 APC
- △ M29C WEASEL
- ▽ M48A1 TANK
- □ CD4 ENGINEER TRACTOR

RELATION OF PULL COEFFICIENT TO LOG (SAND-TRACK MOBILITY NUMBER) FOR PROTOTYPE VEHICLES

20-PERCENT SLIP POINT
AIR-DRY MORTAR SAND

APPENDIX A: INFLUENCE OF SAND STRENGTH PROFILE ON TRACK PERFORMANCE

The Problem

1. For each test reported herein, the sand was prepared such that cone index increased in near-linear fashion to approximately the 30-cm depth (paragraph 6 in the main text). There was concern that track performance might be influenced by soil conditions below the 30-cm depth, particularly in tests with the wide (61.0-cm) track so that special analysis of the sand cone index profile might be required. Several soil mechanics theories indicate that the depth range within which changes in density or soil strength affect the bearing capacity of sand is proportional to the width of the footing--in this case, the track. However, resistance to track motion is provided primarily by sand displacements perpendicular to the width direction. Finally, there does not exist among researchers of the sand-track system general agreement regarding the sand depth(s) of primary importance to track operation.

Solution to the Problem

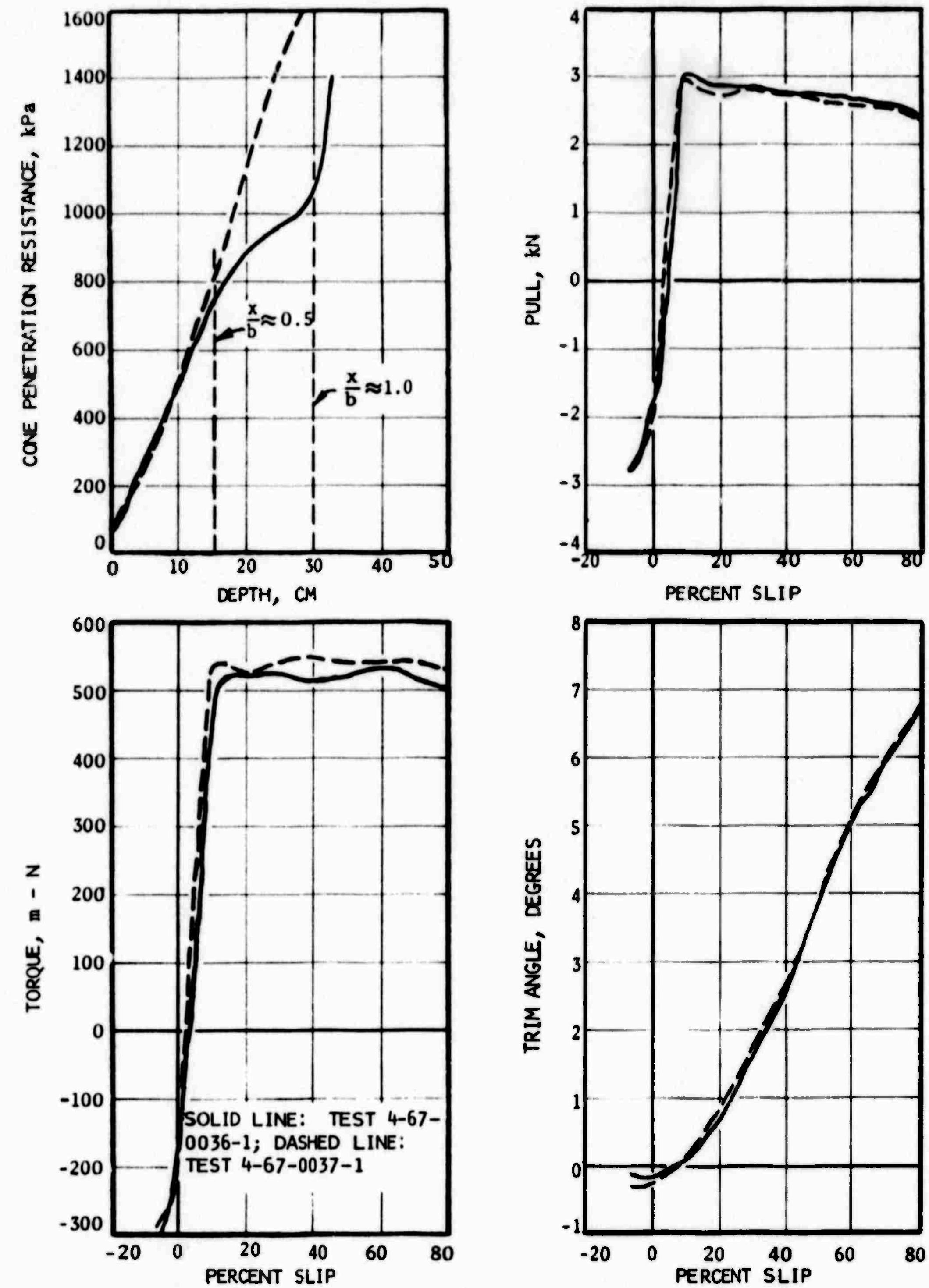
2. In considering whether constructing sand test sections of uniformly increasing strength profiles to only the 30-cm depth is sufficient for the 61.0-cm-wide track, it was assumed that the sand depth of importance is directly related to track width (b). (This assumed, in effect, that bearing capacity is the predominant sand property in relation to track performance.) This hypothesis was tested by conducting eight tests with the 30.5-cm-wide by 121.9-cm-long track. Test conditions included four combination of load and soil strength (4.5- and 6.7-kN design loads with $G \approx 4.4$ MN/m^3, and 13.3- and 26.7-kN design loads with $G \approx 2.1$ MN/m^3), each tested in two types of soil beds--one with soil strength profile linear to a depth x of approximately 30 cm ($x/b \approx 1.0$), the other with profile linear to $x \approx 15$ cm ($x/b \approx 0.5$). The curves in Figure A1 show that pull, torque, and trim angle behaved almost identically within the -10 to +80 percent slip range for two

paired tests, each similar except for their x/b values.

3. For the eight tests, the following tabulation lists values of load, pull, torque (each averaged over the 15 to 50 percent slip range where their values were nearly constant), trim angle at 20 percent slip, and penetration resistance gradient measured within the 0- to 15-cm depth (G_{0-15}).

Test No.	Penetration Resistance Gradient G_{0-15}, MN/m^3	Approximate x/b	Average Values, 15- to 50-% Slip: Load W, N	Pull P, N	Torque M, m-N	Trim Angle at 20% Slip θ_{20}, deg
4-67-0036-1	4.37	0.5	4435	2847	523	1°23'
4-67-0037-1	4.42	1.0	4390	2740	537	0°43'
4-67-0034-1	4.23	0.5	6757	3834	780	0°53'
4-67-0035-1	4.70	1.0	6703	3945	765	0°42'
D-70-0083-1	2.18	0.5	13754	7207	1698	-4°09'
D-70-0021-1	1.99	1.0	13579	6352	1466	-1°19'
D-70-0084-1	2.22	0.5	25041	11412	2875	-4°12'
D-70-0022-1	2.02	1.0	27034	10581	2776	-0°56'

For both pull and torque, only small differences are noted between values of corresponding terms for the paired tests, and these appear to be related to small differences in values of G_{0-15} (e.g. pull and torque values of test 83 are about 14 percent larger than those of test 21; the G_{0-15} value of test 83 is about 10 percent larger than that of test 21). The very small trim angles of the eight tests indicate that this variable had little influence on test results. Cumulatively, the results in Figure A1 and in the above tabulation indicate that the shape of the cone penetration resistance profile below $x/b \approx 0.5$ does not affect track performance significantly. In the main text, analysis of data from subsequent tests with tracks of three widths--15.2, 30.5, and 61.0 cm--in sand test sections of $\approx$30 cm confirmed this observation.



TRACK WIDTH = 30.5 CM; TRACK LENGTH = 121.9 CM; AIR-DRY YUMA SAND
NOMINAL LOAD (BOTH TESTS) = 4.5 kN; NOMINAL G_{0-15} (BOTH TESTS) = 4.4 MN/m^3

Figure A1. Influence of sand strength profile on track performance

APPENDIX B: NOTATION

The following notations are used in the U. S. Army Engineer Waterways Experiment Station soil-track research. Other terms that are used specifically and only once in this report, and are defined in context, are not listed here.

A	Ground contact area of track (usually refers to the product of contact width b times nominal contact length ℓ)
b	Track-ground contact width
c	Soil cohesion
C_s	Average standard soil penetration resistance obtained by penetrating the soil at 3.05 cm/sec with a 30-deg-apex angle right circular cone and dividing the average soil penetration resistance (in newtons) by the base area of the cone (3.23 cm^2) and converting the value to kilopascals
CG	Track center of gravity
d	Horizontal distance from center line of track rear road wheel to track RCG_h, measured with the track on a flat, level surface
dp_b	Distribution of pressure in track road-bogie cylinders
ds	Drive sprocket location
d_w	Diameter of track road wheel
f	Track-soil friction
F	Front
g	Acceleration due to gravity
G	Soil penetration resistance gradient (a subscript with G, e.g. G_{0-15}, denotes the depth of soil in cm that G describes)
h_s	Track-shoe height
ℓ	Nominal track-ground contact length (i.e. contact length on a flat, unyielding surface)
ℓ'	Any particular track dimension pertinent to the description of a given feature of track performance
M	Net torque input at the drive sprocket
n	Exponent in the term $\left(\frac{d}{\ell/2}\right)^n$
N_{BV}	Basic-variable sand-track prediction term
N_s	Sand-track mobility number
P, P_{20}, P_T	Track pull, track pull at 20 percent slip, and track towed force, respectively

p_b Pressure in a given track road-bogie cylinder

r Track drive-sprocket pitch radius

R Rear

RCG_h Horizontal location of track at-rest center of gravity

RCG_v Vertical location of track at-rest center of gravity

S Track slip

s_s Track-shoe spacing

s_w Road-wheel spacing

t_{tb} Index of track-belt tension

TE, TE_{max} Tractive efficiency and maximum tractive efficiency, respectively

th_s Track-shoe thickness

V_a Actual translational velocity of overall track system

V_t Theoretical track translational velocity (equals product $r\omega$)

W Vertical load on the track

W_{max} Load that causes maximum track road-bogie deflection

z, z_F, z_R Sinkage of the track, and sinkage of the track at the front and rear road wheels, respectively

α Angle of approach of the track

β Angle of departure of the track

δ Deflection of a track road bogie measured with the track operating in soil

Δ Deflection of a track road bogie measured with the track on a flat, level, unyielding surface

Δ_{max} Maximum deflection of a track road bogie

γ, γ_d Soil density and soil dry density, respectively

ϕ Angle of internal friction of the soil

η Soil spissitude

θ Track trim angle

ω Angular velocity of the track drive sprocket

In accordance with ER 70-2-3, paragraph 6c(1)(b), dated 15 February 1973, a facsimile catalog card in Library of Congress format is reproduced below.

Turnage, Gerald W
Performance of soils under track loads; Report 3: Track mobility number for coarse-grained soils, by Gerald W. Turnage. Vicksburg, U. S. Army Engineer Waterways Experiment Station, 1976.
1 v. (various pagings) illus. 27 cm. (U. S. Waterways Experiment Station. Technical report M-71-5, Report 3)
Prepared for U. S. Army Materiel Development and Readiness Command, Alexandria, Va., under Project 1G662601AH91, Task A046, Subtask 06.
Includes bibliography.

1. Coarse-grained soils. 2. Dimensional analysis. 3. Mobility. 4. Tracked vehicles. 5. Vehicle performance. I. U. S. Army Materiel Development and Readiness Command. (Series: U. S. Waterways Experiment Station, Vicksburg, Miss. Technical report M-71-5, Report 3)
TA7.W34 no.M-71-5 Report 3